Notes for Partitioning Video

In this video we describe instructional activities that we call Partitioning Activities. Partitioning activities complement the patterning activities described in Finger Patterns, Dot Patterns, and the Single ten-frame. In the patterning activities children use grouping strategies to figure out the total. They build up the quantity by combining smaller quantities. Partitioning, on the other hand, involves breaking a quantity down into parts. For example, you can partition 7 by splitting it up into 4 and 3. We use the label partitioning because it describes the activity of the child, whether physical or mental that results in a partition. Children's understanding of partitioning numbers develops in tandem with patterning. Here we describe several activities that have partitioning as their express purpose. In each case, children start with a specified quantity and their task is to break it into smaller quantities.

The purpose of these activities is for children to begin to develop:

- Partitions of numbers to 10.
- Notions that the partitions of a number are related to the partitions that make up another number. For example, that the partitions of 7 can be thought of in relationship to the partitions of 6.

Here we describe 3 activities that have partitioning as their express purpose. They are:

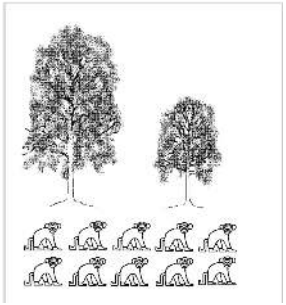
- Monkeys in the Trees
- Double-Decker Bus
- All the Ways

In each case, children start with a specified quantity and their task is to partition it or break it into smaller quantities. These instructional activities are intended for use along with and intermingled with the patterning activities.

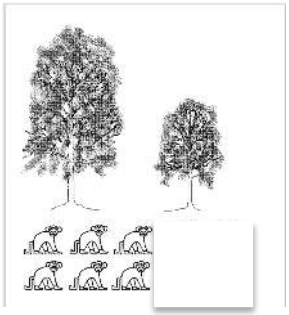
Monkeys in the Trees

The first activity is one that we call "Monkeys in the trees." This title is based on the scenario that underlies this activity. We will give an example of the activity, then describe its purpose, and finally talk about what children do.

In this whole-class, visual activity the teacher uses the provided picture to show the two trees and some of the monkeys. The picture provided shows 10 monkeys so the task can be about any number 10 and below.



The teacher simply shows the number she wants the task to be about for that lesson. For example, if the teacher wants the task to be about partitioning the number 6, she shows 6 monkeys. It's easy to do. Just cover up 4 monkeys so only six are visible. To explain this activity we'll use the example of 6. So the teacher shows the picture with 6 monkeys visible.



Initially, the teacher explains the scenario. The scenario is of monkeys that like to play in trees. There are two trees the monkeys can play in, a big tree and a little tree. It is important to emphasize that all of the monkeys are in one tree or the other. None are on the ground. The question is:

How many monkeys might there be in the big tree and how many in the little tree?

The children's task is to come up with different ways to answer that question. The teacher records the children's responses in some way. One useful way is to use a table to record the children's solutions.

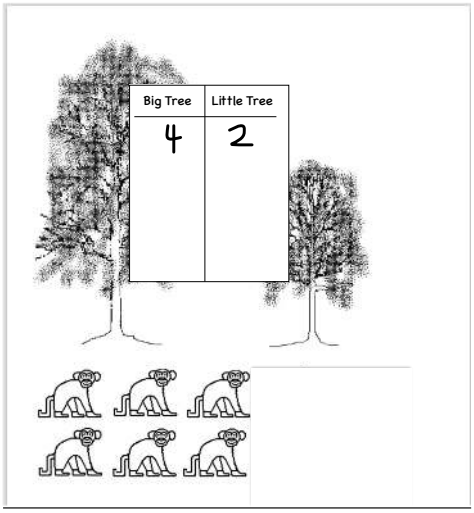
One feature of this activity is that the visual material is not flashed but is left visible for the children to see throughout. Another feature is that, unlike the dot pattern and ten-frame activities where the teacher uses chips that can be moved around during the discussion to provide clarification or elaboration, here the pictures of the monkeys are fixed on the page. The teacher cannot "move" the monkeys into the trees during the discussion. This feature is deliberate because it requires children to reason more abstractly about the visual material. Children can only think about which tree the monkeys are in but they cannot actually see the monkeys in those trees.

So what do children do? Typically teachers use this activity after children have already worked with patterning activities for several weeks and therefore have already started to develop notions of partitioning numbers. As a result, a number of children will offer ways the monkeys will be in the trees based on their already developed understandings. At the same time, other children use their finger patterns to generate partitions.

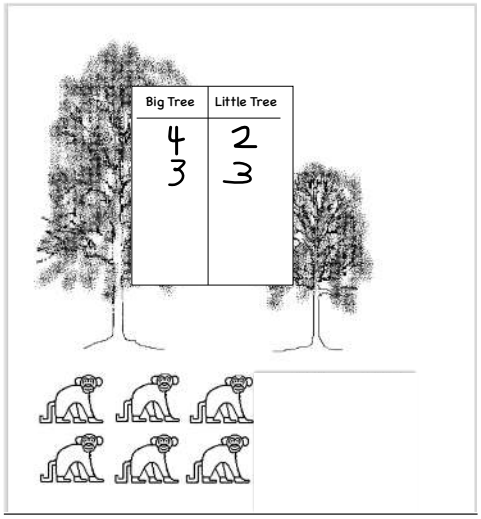
Let's take a look at Katrina's solution. She puts up four fingers on one hand, while counting, "1, 2, 3, 4," and continues counting, "5, 6," while putting up fingers on the other hand. Katrina then looks at her hands to figure out that 4 monkeys can be in the big tree and 2 in the little tree.

While on some activities the focus is on discussing how the student made the 6, this is not the focus of this activity. The goal is to provide opportunities for the class to reflect on the ways that 6 can be partitioned. To this end, the teacher might say something like, "Katrina

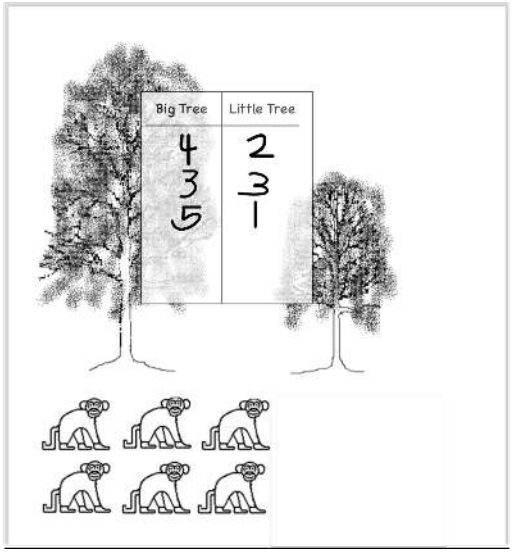
said that there are 4 in the big tree and 2 in the little tree.“ She might then draw a table to record this result and then continue by saying something like this. “Is that 6? Did anybody else think about 4 in the big tree and 2 in the little tree?”



Another child in this class, Sam, might just know a finger pattern for 6 and use that to generate a way the monkeys can be in the two trees. He might hold up 3 fingers on each hand and say, “Oh, I know that 3 and 3 is 6.” The teacher might say, “How many people agree with Sam that 3 and 3 will be 6? That’s a double,” and record his result.

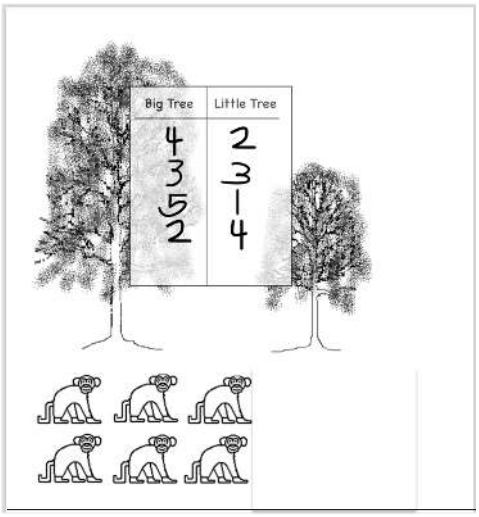


Other children by now already just know some combinations that make 6. For example, James might say, “I know 5 and 1 make 6, so put 5 in the big tree and 1 in the little tree.” Again, the teacher records the contribution and then attempts to draw others into the conversation. “Morgan, do you agree with James? Will 5 and 1 more make 6?”



As these example show, children at widely differing number conceptions can participate meaningfully. Katrina counted. Sam already had a finger patterns for 6. And James just knew a combination. An underlying principle of our inquiry approach to mathematics instruction is that all children should have personally meaningful ways to participate in every task posed.

One important mathematical principle inevitably comes up in the discussion of this activity, namely that you can change the order of the monkeys. For example, Thomas might say, "Just switch them around. Put 2 in the big tree and 4 in the little tree."



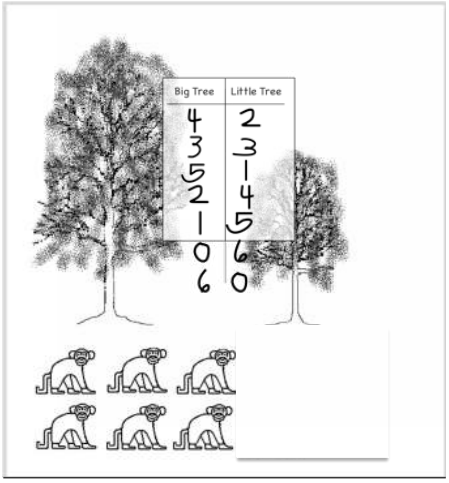
Children often express amusement at this "discovery," indicating that they come to realize it as a general principle that applies to any pair of numbers. For example, once one child has suggested switching a pair of numbers around, others often make similar suggestions, such as, "Put 5 in the little tree and 1 in the big tree."

This example illustrates how important mathematical ways of reasoning emerge from apparently simple instructional tasks. In this case, the teacher is not the one that introduced the idea to students. Rather, the students become aware of it on their own, representing an important advance in their understanding of number relationships.
It is also important to note that this discovery will not be made by all students at the same time. Nevertheless, the teacher can call attention to this issue when it arises, perhaps by asking, the class, "Thomas said if we know 4 are in the big tree and 2 are in the small tree, we can also have 2 in the big tree and 4 in the small tree. What do you understand about what Thomas is saying?" In this way, students that did not make sense of what Thomas said initially, have an opportunity to further reflect on it.

This instructional activity also reveals that not all students know that if, for example, 4 and 2 is given, that 4 and 3 or 4 and any number other than 2, will not give a total of 6 monkeys. For example, Carly might offer a solution of 4 in the big tree and 3 in the little tree. In keeping with our approach that the teacher does not evaluate every child's comments as correct or incorrect, we recommend that in, this case, the teacher simply records the incorrect answer in the same way as the correct answers. She might repeat what Carly said, "Carly said to put 4 monkeys in the big tree and 3 in the little tree."

By this time children are beginning to understand that it is their responsibility to ask questions or raise challenges if they disagree with what someone has offered. In the case of 4 and 3 given above, someone will disagree, "Because 4 and 3 is 7 and not 6." Or, another child might say, "If 4 and 2 is 6 then 4 and 3 can't be also 6, because that's one more." This argument is quite sophisticated and therefore is worth being acknowledged and reiterated by the teacher.

We have talked about how the teacher initiates the Monkeys in the Trees activity, various ways children might generate partitions, and important mathematical principles that typically come up in the discussion. Another important issue arises as the teacher continues to ask, "Who has another way?" even after all the possible ways have already been given. At this point the table might look like this.



In doing so, the teacher shifts the children's mental activity from generating a single partition of the number to considering the set of partitions as a whole. In our experience, most children continue to try to come up with more ways, but usually someone will suggest that there are no other ways; that all of the ways have already been given. Now the question becomes, "How do we know we have all the ways? How can we be sure?" This question is of a completely different nature and requires a different type of cognitive activity from generating the partitions themselves. This question is a higher-level question and one that we would expect not all children can answer at this stage. In fact, typically, the majority of the children decide that there are no more ways because they cannot think of any others. Several children may attempt to explain other ways to reason about how they know for sure there are no more ways. However, often this question remains unresolved for the class. We urge teachers to not provide a resolution to this question themselves but rather wait for it to emerge from the students. The rationale for this suggestion is that for reasoning to make sense to students it must arise from their own conceptual understanding. Remember that the goal of this activity, as with all of our inquiry activities, is that students say and do things that make sense to them and not simply repeat statements or procedures that have been shown to them.

You will notice, for example, that the numbers shown in the table were recorded in the order in which they were given by the class. The teacher made no attempt to put them in any specific order or to indicate how many combinations there are. Further, when she drew the table initially she did not draw in exactly the number of lines needed for a complete set of combinations. These actions were deliberate. This activity is not about being able to complete an ordered set of numbers that combine to make 6, quickly and easily. While it is very tempting to put the number pairs in order, we have found that when the teacher does so, students often learn to replicate the number combinations mindlessly and no longer think about the number relationships. That means that this activity is no longer useful as a learning activity. It is important to clarify that we do not intend that after engaging in this activity children will just know all of the partitions of the number used in the task or know how many partitions there are for a particular number. This is in sharp contrast to many traditional instructional materials that do have that as a goal. The Monkeys in the Trees activity is only one of many activities that lead to children's development of partitions. In other words, we do not intend that this activity be repeated over and over again until children can list all partitions easily. It might be used only a few times and with only some of the numbers to 10 as choices.

We have already noted that the question of how can we be sure we have all the ways is a higher-level question than the question of generating a partition of the number. It involves thinking and reasoning about the entire set of partitions of the number. The teacher may decide to pose similar higher-level questions such as:

Will there be more ways or fewer ways if there were only 3 monkeys playing in the trees?

Or

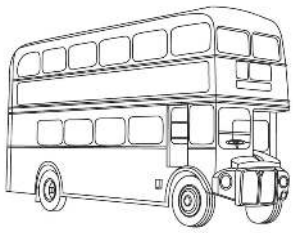
Will there be more ways or fewer ways if there are 10 monkeys playing in the trees?

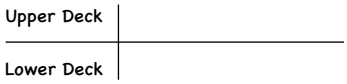
These questions are less complex than the question of how many partitions there are of a particular number. Most students have a way to think about these less complex questions.

We have used the task with six monkeys to illustrate the features of the task, how it might be posed, how children engage in the task, productive teacher actions, and how the task can lead to higher level questions and higher level reasoning. Another way the teacher can promote children's advancement in partitioning is to be deliberate in posing tasks in sequence. For example, if the teacher poses the task using 7 monkeys (or 5 monkeys) after the task with 6 monkeys, it is likely that at least some children will relate the partitions for 7 (or for 5) to the partitions for 6. A child might say that since for 6 monkeys there could be 3 in each tree, for 7 monkeys there could be 3 in one tree and 4 in the other. Such apparently simple teacher moves have huge implications for children's learning. For this reason, careful teacher planning is essential.

Double-Decker Bus

Another partitioning activity is the Double Decker Bus. This activity is much like the Monkeys in the Trees but instead uses the scenario of a double-decker bus. The number of people on the bus is specified and the children's task is to figure out how many people might be on the upper deck of the bus and how many might be on the lower deck. In this activity a picture of a double-decker bus is shown but there are no images that represent people. In this sense the activity is more abstract than the monkeys activity.

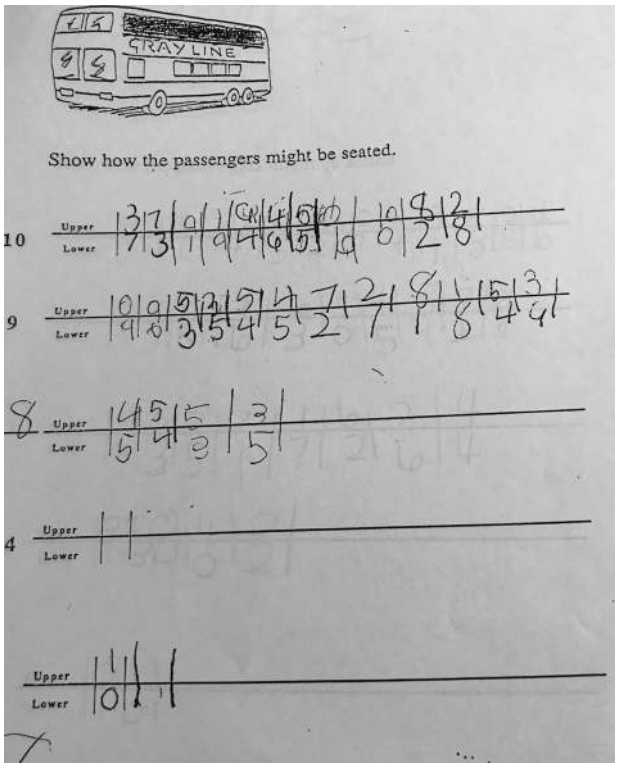


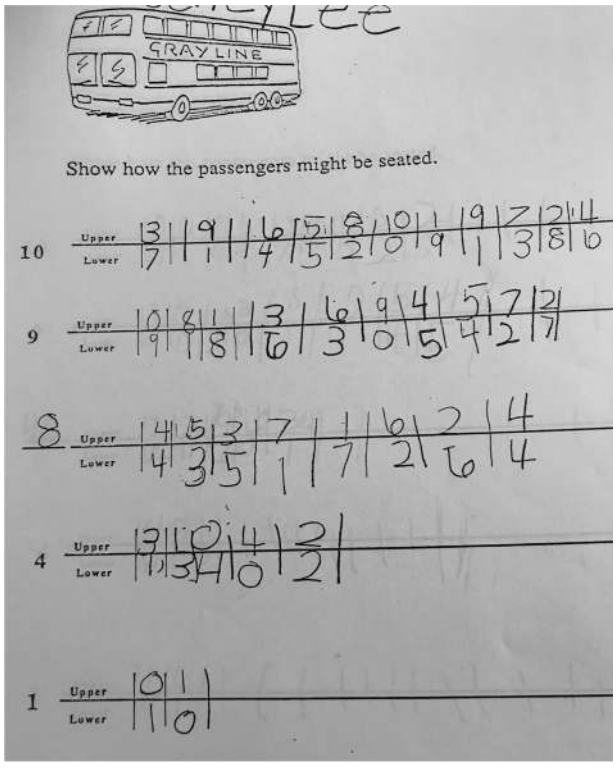


This activity can be used as a whole class activity where the teacher records the partitions students suggest or it can be used for partner or individual work. In the latter cases, the task is for the pair of children or the individual child to write down as many ways the people can be on the bus as they can think of. A culminating discussion of what they have written down is important for the activity to promote the learning intended. Through the discussion, children have an opportunity to hear the partitions others have listed and how other children have reasoned.

The teacher may choose to use this activity only once or twice or not at all. Its primary purpose is as a link to the next instructional sequence, Structuring Numbers. The Structuring Numbers sequence is based on the scenario of a double-decker bus. Teachers that choose to not use this activity here can easily introduce the double-decker bus scenario when needed later.

Here you see samples of students' work for an adaptation of the double-decker task that was used early in second grade. Students worked in pairs to complete the tasks, which were then discussed in a whole-class setting.

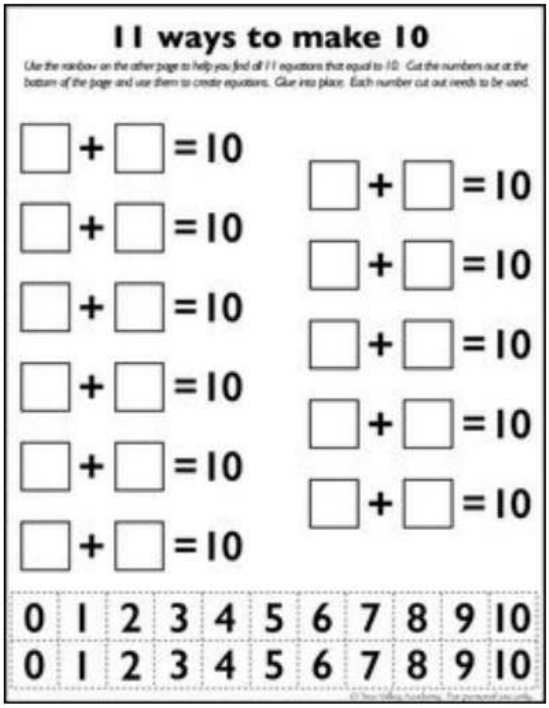




Of the five tasks on the page, four were prescribed. The fifth task, shown in the middle of the page, was left blank so that students could choose the number they wanted to use. Children's choices included 8, 11, 20 and 5. Both of the examples shown here have 8 as the choice. These samples show the various ways children thought about partitioning a number. Some were somewhat systematic. Some were more random. Some included errors. Notice, for example in the sample page on the left that partitions for 9 include 5 and 3, and 3 and 5 immediately followed by 5 and 4, and 4 and 5, and also 3 and 6. And on this same sample page, partitions for 8 include 4 and 5, 5 and 4, 5 and 3 and 3 and 5. None of the solutions include all of the possible partitions. By contrast, on the sample page on the right there are no errors and most of the partitions for each number are given. We reiterate that this page was used with second graders. We do not advocate using this page of tasks in grade 1 but rather using the double-decker bus task in the whole class setting, or, if used as partner or individual work, posing just one number at a time, followed by a discussion.

All The Ways

Yet another partitioning activity is one we call "All the Ways." This activity grew out of observing students complete a typical textbook task that proved unproductive for learning. We first show the textbook task to clarify the contrast in the tasks.



11 ways to make 10

Use the rainbow on the other page to help you find all 11 equations that equal to 10. Cut the numbers out at the bottom of the page and use them to create equations. Glue into place. Each number cut out needs to be used.

□ + □ = 10
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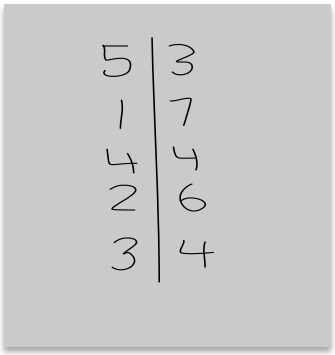
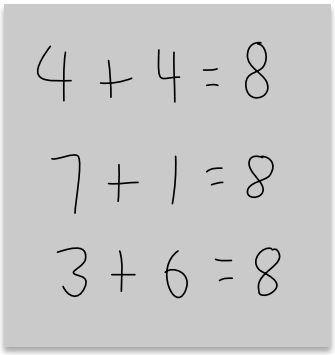
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9 10

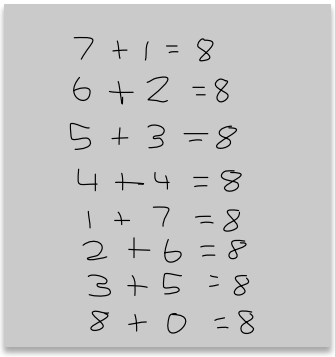
On this typical textbook worksheet, the student is instructed to "cut out the numbers and then glue them into the boxes" in order to "make ten." Besides the fact that cutting and gluing takes a lot of time away from engaging in meaningful mathematical activity, we still question the "11 ways to make 10" that is the title of this page. This title is telling student that there are exactly 11 ways using two numbers in which you can make 10. We have seen children successfully complete the task and yet not be able to generate the partitions on their own in a non-structured setting. Experience with tasks such as the one above led us to opening up the activity in the following way.

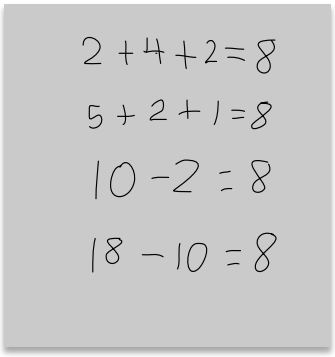
We illustrate it with this example. Each child has a piece of paper and their task is to write all of the combinations they can think of for a given number, in this case, the number 8. Write all of the ways you can to make 8.

Experience shows that some children write number sentences, some make a chart with columns. Some children will generate all possible combinations using two numbers. Some children may even have the combinations listed systematically. Some will generate only one or two combinations. Other children will go on to use more than one summand and some will use subtraction as well. Children that complete the task quickly can be given another number to work on. Below we show samples of first graders' work.









Notice the similarity of this task to the double-decker bus task. There are several important differences. One difference is that here the task is posed without a context. The other difference is that here the task is much more open-ended, as we have just indicated. The double-decker bus activity is limited to partitioning the number into two numbers that when added yield the given number.

Teachers have found this activity to be useful as an assessment tool. Through this activity, teachers can gauge the child's ability to generate partitions of a number, including the number and accuracy of partitions they generate and whether or not they are generated systematically. For this reason we recommend not having a class discussion after this activity. Instead, the teacher can send the children's work home for parents to see or post them on a class bulletin board.

Summary

In this video, we have described three instructional activities that are specifically designed to promote partitioning. Each of these, *Monkeys in the Trees*, *Double-Decker Bus*, and *All The Ways*, involve breaking a number apart that is, partitioning it into one or more smaller numbers. As we have already noted, children begin developing partitioning concepts when they engage in patterning activities, such as finger pattern, dot pattern, and single ten-frame activities. With this in mind, we recommend that teachers begin to use the partitioning activities around the fifth week of the school year, while they are still making heavy use of patterning activities.

It is important to remember that all of these activities have as their purpose promoting conceptual development of children's world of number. The purpose is not that children master each of these activities. Teachers that use these activities over and over until their students "get good at them" are missing the intention. Our goal is not that children learn how to "do" these activities. Rather, the intention is that through engaging in these activities, along with patterning activities, children begin to build a conceptual foundation for reasoning about quantity. For that reason we recommend using these activities only a few times, as we have described earlier, and as listed here. We encourage teachers to keep moving forward.

Timeline

We include a timeline here for your reference. As we have noted above and as the timeline shows, we recommend that teachers begin to use the partitioning activities around the fifth week of the school year, while they are still making heavy use of patterning activities.

SUGGESTED TIMELINE FOR FIRST 6 WEEKS OF GRADE 1