Coordinating Units Notes for Video Group 2
E.Yackel

Notes for Coordinating Units Video Group 2

Part II: Becoming Efficient With Reasoning About Sums And Differences By Coordinating Units.

Part II of the Coordinating Units of Tens and Ones instructional sequence is designed to develop children's becoming efficient with reasoning about sums and differences by coordinating units.

In addition to an initial introductory activity, there are four whole class activities and one partner activity in Part II. They are:

- Reasoning using Unifix cubes
- Anticipation activity
- Reasoning without Unifix cubes
- Imagination bingo
- Partner activities using Unifix cubes

Activities in this part involve changing the quantity.
Examples of tasks are:

- Mrs. Wright has 26 cherry candies. She makes 38 more. How many cherry candies does she have now?
- Mrs. Wright has 54 lemon candies. She sells 18 of them. How many lemon candies does she have now?

The purpose of this part of the sequence is to foster students' development of the conceptual basis for solving additive problems involving numbers to 100 by coordinating units of ten with units of one.

Notice that the difference between Parts I and II of this sequence is analogous to the difference between Parts I and II of the Structuring Numbers sequence. In both sequences, Part I involves a single quantity whereas in Part II the initial quantity is changed. In Part II of the Structuring Numbers sequence people get on or off the bus. In Part II of the Coordinating Units sequence Mrs. Wright makes more candies or sells some.

Further, the activities in this part of the Coordinating Units sequence follow a similar progression as those in Part II of the Structuring Numbers sequence. In both cases, initially students reason by using physical materials and progress to reasoning using mental imagery. Here the physical materials are Unifix cubes that students use as pretend candies. The tasks evolve to those where the **teacher** uses cubes to show the initial quantity and students **imagine** how they would solve the problem using cubes. The anticipation activity is an intermediary activity, as we will explain later.

In addition to whole class activities, this part of the sequence also includes partner work where students use cubes to act out tasks within Mrs. Wright's candy shop.

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Initial Introductory Activity

In Part 1 of this sequence children used Unifix cubes as pretend candies to pack up individual loose candies (pieces) into rolls. In this part of the sequence they will again use Unifix cubes, this time to act out tasks where Mrs. Wright makes more candies or sells some. Children have a supply of Unifix cubes to use to solve the tasks. Either they work individually or with a partner, sharing one bag of cubes. So, children have to understand how to use the cubes as props to act out the story. In our experience there are always some children that do not understand how to do so. This initial introductory activity has as its purpose to clarify this issue.

For example, suppose the task posed is:

> Mrs. Wright has 26 orange candies. She makes 18 more. How many orange candies does she have now?

Children are expected to use their bag of Unifix cubes as pretend candies to act out the story. How do children use the cubes? First they get out 26 candies from their bag of Unifix cubes. These are the 26 candies Mrs. Wright has. Where do the other 18 candies come from? They also come from the child's bag of cubes, from his supply of pretend candies. Most children understand this and get out 18 more candies from their bag to act out this story. But interestingly, there are always some children in the class that do not understand from where they should take the 18 candies. Some take them from the 26 they got out initially. In other words, these children need to understand how to use the Unifix cubes to act out the story.

One way to clarify how to use the cubes to act out the story is to act out one or two tasks as a whole class. In this case, the emphasis is on how to use the cubes rather than on the resulting quantity. After getting out the initial 26, the teacher can ask the class, "From where can we get 18 more candies?" In the discussion, children will talk about how their bag of cubes is essentially a set of props for acting out the story. This discussion may need to take place with several different tasks on several days to ensure that all students understand how their actions with the cubes "show" what happens in the story. In our experience, teachers typically show great creativity in figuring out how to make sure children understand how to use their cubes to act out the story.

Likewise, children need to understand how to use the cubes to act out a story where a customer buys some candies. For example, suppose the task is:

> Mrs. Wright has 42 lemon candies and a customer buys 29 of them. How many lemon candies does Mrs. Wright have after she sells these?

How do children use the cubes to act out this story? First they get 42 candies from their bag of Unifix cubes. Next, to show that a customer buys 29, they have to take 29 cubes away from those 42. Most children understand this and act out the story appropriately without any difficulty. However, there are always some children that do not understand that the 29 should be removed from the 42.

Again, one way to clarify the story for these children is to act out one or two tasks as a whole class. In this case, it is useful to have one student act as Mrs. Wright or one of her workers and another to act as the customer. Doing so typically serves to clarify how to use the cubes for those students that are unsure. And again, in this case, the emphasis is on how to use the cubes to act out the story rather than on the resulting quantity. As we said for the tasks of making more candies, teachers typically show great creativity in figuring out how to ensure that their students understand how to act out the story.

Reasoning About Sums And Differences Using Unifix Cubes

This instructional activity **Reasoning about sums and differences using Unifix cubes** will be used for many lessons. It is a whole class activity. Children work individually or with a partner using their supply of Unifix cubes to act out the story the task describes and to figure out how many candies Mrs. Wright has now. We repeat the examples we have given before.

Example of making more:

> Mrs. Wright has 26 cherry candies. She makes 38 more. How many cherry candies does she have now?

Example of selling some to a customer:

> Mrs. Wright has 54 lemon candies. She sells 18 of them. How many lemon candies does she have now?

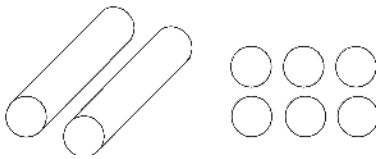
Getting out the initial quantity

Before we discuss what children do to solve tasks like these and how their learning advances over time, I want to discuss only the first part of acting out the task, getting out the initial quantity.

There are several different ways the teacher might use to pose tasks like these. Here's what I mean. To pose the task:

> Mrs. Wright has 26 cherry candies. She makes 38 more cherry candies. How many cherry candies does she have now?

the teacher might show (not flash) the initial quantity using visual materials as shown here



Makes 38 more

and then say, “These are the cherry candies Mrs. Wright has. She makes 38 more. How many does she have now? Use your pretend candies to show this story and to figure out how many Mrs. Wright will have after she made the 38 more.” In this case the teacher leaves the image visible for students to see and writes, “Makes 38 more,” so students can remember the task. Alternatively, the teacher might pose the task entirely orally, but write the quantities for children to see so they can remember the task.

When the teacher poses the task using a visual image to show the initial quantity what we see is that all of the students get out their pretend candies in the same way, using rolls and pieces, as shown. However, when the teacher poses the task entirely orally, children do not necessarily show the initial quantity as rolls and pieces. The way they get out the initial quantity is often very revealing and can help the teacher understand something about the children’s current understandings of tens and ones. We consider several examples of students' activity when the task is posed orally, using examples taken from classrooms we have visited.

The first step in acting out this task is to get out 26 candies. This will be relatively easy by now for the majority of students. They will get out 2 rolls (2 sticks of 10) and 6 pieces, (6 individual cubes). Some will get the 6 pieces by breaking off 6 cubes, one by one, from a stick of ten, while counting, “1, 2, 3, 4, 5, 6.” Others might get out the 6 pieces by taking a ten stick and breaking off a group of 4 cubes, leaving a group of 6 cubes, snapped together. We will come back to say more about this later.

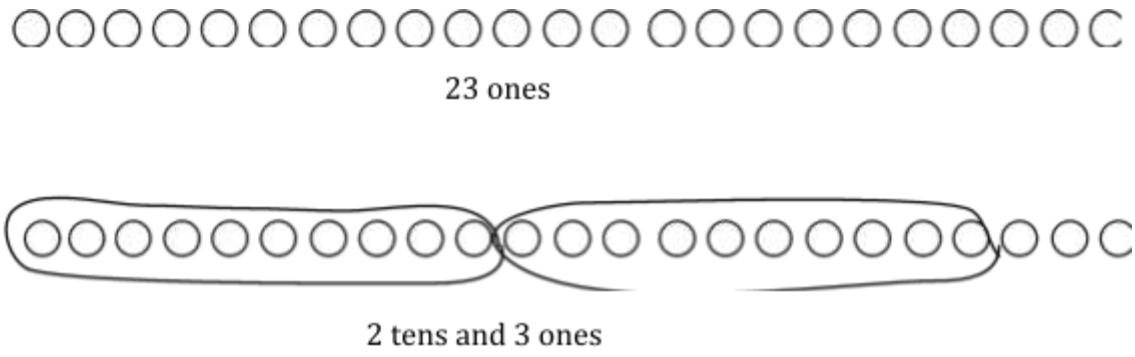
Some of our least sophisticated students will use a different approach to get out 26 candies. Sam gets a stick of 10 out of the supply of cubes and counts from one, by ones, 1, 2, 3, breaking off a cube with each count until he gets to the end of the stick. He is at 10 in his count. Then he gets out another stick of ten and continues counting by ones 11, 12, 13, 14, again breaking off a cube with each count until he gets to the end of this next stick. He is at 20 in his count. He then gets out yet another stick of ten and continues counting by ones 21, 22, 23, 24, 25, 26, once again breaking off a cube with each count. Sam now has a pile of 26 individual cubes. He has 26 candies but they are all in individual pieces. He has 26 ONES.

Here’s another approach we have seen from our less sophisticated students. To get out the initial 26, Danielle gets a stick of 10 out of the supply of cubes and counts from one, by ones 1, 2, 3, all the way up 10 but she does not break the cubes apart. Instead she leaves them snapped together. She gets out another ten stick and continues counting by ones 11, 12, 13, 14, all the way up to 20. She snaps these 10 cubes onto the 10 she already had, resulting in a long stick of 20. She then gets out yet another stick of 10 and continues counting by ones, 21, 22, 23, 24, 25, 26 and snaps these 6 cubes onto the long stick of 20 to yield a very long stick of 26 cubes. Danielle has a group of 26. She has ONE 26.

Both Sam and Danielle are thinking of 26 as 26 ones. The difference is that Sam has a pile of 26 ones whereas Danielle has created a single entity, “a 26.” Neither child has organized the 26 candies into 2 tens and 6 ones, even though the Unifix cubes they have in their bag had

already been packed into sticks of 10. In other words, they were unable to use the tens structure imposed on the cubes in their bags to help them get out 26 cubes. I am confident that if we were to ask Sam or Danielle how many cubes are in one ten stick of cubes in their bag of supplies both of them would answer, "10." But we can see from what they did that they are not able to use the information that there are 10 in one stick. They are unable to conceptualize 26 as 2 tens and 6 ones. For them, 26 is 26 ones, or a single entity, "a 26."

Teachers will notice these vast differences in the way the students work with the cubes. These differences reflect their current understandings of ten. Let me say more. Children's concept of number begins in grades K and 1 by thinking of a number as "one more." That is, 5 is one more than 4, 6 is one more than 5, 11 is one more than 10, 12 is one more than 11, 24 is one more than 23, etc. In this early conception of number, ten does not yet hold a preferential status. For example, at this stage the child does not think of 12 as 10 and 2 more or of 23 as two 10's and 3 ones. So, a huge conceptual development that needs to take place is for the child to "overlay" a tens structure on the existing ones structure. One step in this development is for the child to be able to think of number in terms of tens and ones, for example, to think of 12 as one 10 and 2 ones or of 23 as two 10's and 3 ones.



Overlaying a tens structure on a ones structure

This is where the initial scenario of the candy shop comes in. In an attempt to be efficient in counting out candies, Mrs. Wright packs candies into rolls, packing 10 individual pieces into each roll. At first glance, conceptualizing any quantity in terms of rolls and pieces appears to be a relatively easy development for children to make. And it is one of the advances most children make as a result of Part I of the Coordinating Units instructional sequence. In fact, as a result of working with the activities in Part I of the sequence most of the children will put out 2 rolls (sticks of ten) and 6 pieces if you ask them to show 26. Almost no one will still count the cubes by ones to get 26. However, when we see children able to get out 26 using 2 rolls and 6 pieces, we can be deceived into believing that they now have a true understanding of place value, more specifically of coordinating units of tens and ones. It's when we watch what happens next that we can discern the differences in understanding.

What do children do?

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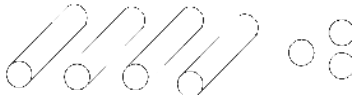
The next two examples are among those I have witnessed across a variety of classrooms as children work with tasks of making more or selling some candies. These examples are illustrative of the ways children engage with the tasks and reveal the wide range of understandings children in any class have at any time.

Example 1:

> Mrs. Wright has this many cherry candies. She makes 19 more. How many cherry candies does she have now?



The teacher posed this task by showing the initial quantity as a visual image, as shown here. This is what Joe and Michael did. They got out 4 rolls (4 sticks of 10) and 3 pieces. Then they got out 19 candies, one by one, breaking off one at a time from the ten sticks that remained in their bag of cubes. In doing so, they broke apart an entire ten stick (a roll) and then continued on using another one, counting as they said, "I, 2, 3, 4, … 19." The 19 pieces were lying on the desk a bit apart from the original 3 pieces. Here you see a picture of what it looked like.

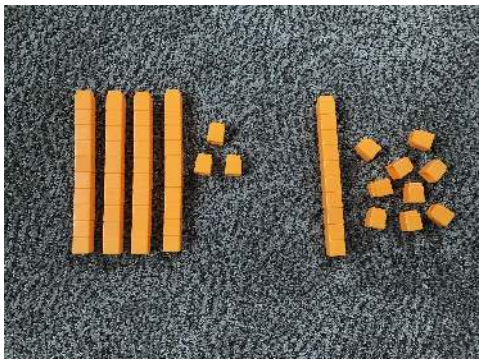
Cubes Joe and Michael got out to solve the task

Next, they counted out 10 pieces from the 19 and packed them into a roll. Then they counted out the remaining 9 pieces (from the pile of 19), took one piece from the original 3 pieces, and packed up another roll. Then they looked at what was lying on the desk. They now had 6 rolls and 2 pieces, and gave their answer as 62.

Notice that Joe and Michael had no difficulty thinking of the final result as 62, based on the 6 rolls and 2 pieces. Also, even though they weren't asked, I am convinced they would have been able to say that the initial configuration represented 43 candies. Nevertheless, when they wanted to get 19 more candies, they did not get out 1 roll and 9 pieces. They got out 19 individual pieces. In other words, these students were not thinking of 19 as one 10 and 9 more, as one roll and 9 pieces.

I interpret this to mean that Joe and Michael are in a sort of transitional stage. When they see a single quantity that is shown in tens and ones, such as the quantity they had at the end of the problem, they are able to say how many it is. However, when they are in the midst of working with quantities, those quantities are still conceptualized only as ones. Nineteen is 19 ones. This way of operating is especially likely when the initial quantity the children have is not a full decade, e.g. not something like 40. In this example the initial quantity was 43. I will say more about this later.

By contrast, other students got out 4 rolls and 3 pieces, then got out 19 as one roll and 9 pieces. Some of these children created a pile of 9 ones while others had the 9 cubes snapped together. Most likely those that created a pile of 9 ones broke off one at a time while counting up to 9, while those that had the 9 snapped together most likely took a ten stick (a roll) and broke off one to leave 9. Again we can see the differences in the children's concepts. Those that left the 9 snapped together were thinking of 9 as one less than 10. The others were thinking of 9 as 1, 2, 3, 4, 5, 6, 7, 8, 9. In any event all of these students now had 5 rolls.



19 as 1 roll and 9 individual pieces

19 as 1 roll and 9 pieces snapped together

Those that made a pile of 9 ones now had to somehow count out a total of 10 cubes and pack up a roll. They might use the 9 and 1 from the 3, use the 3 and 7 from the pile of 9, or if they had pushed all of the ones together, they had to count out 10 from the total pile of ones. In any event, they now had 6 rolls and 2 pieces. On the other hand, those that had the 9 ones snapped together had only to take one from the 3 to easily get another roll.
As children solve tasks like this repeatedly, two ways of using the cubes emerge as most common. We will come back to this point later.

Now let's consider an example where some candies are sold.
Example 2:

> Mrs. Wright had 34 peppermint candies. She sold 18 of them. How many does she have now?

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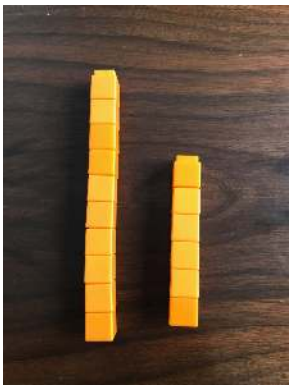
I was standing next to Amber and Tiffany as they solved this task. They acted out the story like this. They easily got out 34 as 3 rolls and 4 pieces. Then Amber gave 18 to me, since I said I wanted to be the customer. Here is how she did it. She gave me the 4 pieces, counting 1, 2, 3, 4 and then continued giving me one cube at a time, breaking cubes off of two ten sticks (rolls), while continuing to count 5, 6, 7, 8, all the way to 18. At that point I had 18 individual cubes in my hand. She had 1 roll 6 cubes left, snapped together, as shown here.



Amber's final result

When I asked Tiffany to act out the story she did the exact same thing.

My interpretation of what is happening here is that, at this time, Amber and Tiffany were unable to think of 18 as comprised of tens and ones, **if they started out by thinking of four ones**. If we simply asked them to show 18, I'm sure they would put out one roll and 8 pieces. However, in this case they each began by giving me 4 pieces. So they were now in the position of having to think about the "rest of the 18." And the only way they could do so was to think in terms of ones. Another way to say it is that they were unable to think of 18 as 4, 10, and 4 more, or even as 4, 4 more, and ten. In interviews we use in our research projects, we have tasks that demonstrate this same inability to think in terms of tens and ones when the child begins with some, but not all, of the ones.

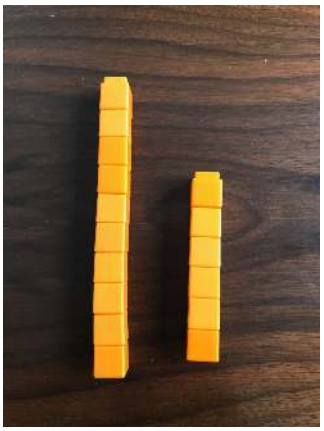
Another thing this way of reasoning suggests is that for Amber and Tiffany, 34 is made up of 3 tens and 4 ones **and the ones are the last part of the 34**. Therefore, when they wanted to take away, they were taking away from 34. They took away **the 34, the 33, the 32, the 31, the 30**, etc. They were not able to think of taking away 18 in parts, such as to take 10 away from "the 30" in the 34, 4 from "the 4" in the 34, etc.

Others solved this problem by taking away a roll (for the 10 in 18) and then taking away 8 more. Most used the approach a student Destiny used, which was to take away the 4 pieces and 4 from a roll. Even here there were differences. Some took away the 4 pieces and then just knew they needed to take away 4 more so broke 4 off of a roll, leaving 6 snapped together. Others took away the 4 pieces and then counted on while breaking off additional

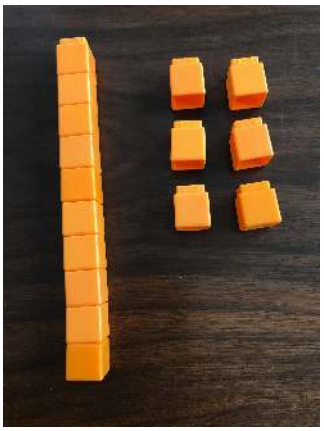
pieces from another roll, while saying, “5, 6, 7, 8.” When asked, those children that did this had no idea how many they had taken from the additional roll. When they were asked, “So how many did you take off of this roll?” they didn’t know. After looking at the 6 cubes that remained and that were snapped together, one of the children said, “Four.” In other words, after the fact, she was able to figure out that it must have been 4, but because of the way she actually carried out the task she didn’t know at the time that she had taken off four.
Still other children, like Andrea, took away 1 roll, 8 pieces from another roll, leaving 1 roll, the 2 snapped together, and the 4 pieces they had initially.

In discussing examples 1 and 2 we said that children sometimes break cubes off of a roll and leave the remaining cubes snapped together. Now when Mrs. Wright breaks open a roll in her candy shop, she will end up with 10 individual pieces. There will be no “chunks” of 6 or of other amounts, left stuck together. So the children’s actions do not exactly mimic what happens in the scenario. Some teachers have asked us about this discrepancy and have questioned whether they should require their students to follow the scenario exactly so that when they want to take pieces from a roll they should begin by breaking the roll into 10 individual pieces and then go from there. Our answer to this is a definitive **no**. Teachers should not require students to break a roll into 10 individual pieces. The reason is this. Remember the purpose of this instructional sequence is to foster children’s coordination of tens and ones so that they can eventually reason entirely quantitatively. The scenario is one tool to get to that goal. But the ultimate goal is that children can reason about tens and ones quantitatively. The goal is not that they mimic the actions of the candy shop. When children leave cubes snapped together they often are able to think of them as a group, for example, they may be thinking that if I take 4 from 10, I have 6 left. In doing so, they are moving away from thinking of everything in terms of ones, thinking one by one, and are thinking in groupings that comprise 10. Insisting that children break apart a roll into 10 individual pieces often puts them in the position of having to count everything by ones—just the opposite of what we are trying to foster.

Similarly, when children get out the initial quantity they might leave the individual pieces snapped together. For example, if the initial quantity is 16, children might get out 1 roll and then break 4 cubes off of another roll, leaving 6 snapped together. They might do this even if the teacher poses the task by showing the initial quantity visually as 1 roll and 6 individual pieces. Here again, having the 6 pieces snapped together can be an advantage for the child in figuring out how many there are in all after more are made because he can take advantage of number combinations he knows as he packs up another roll.



16 as one roll and 6 pieces snapped together



16 as one roll and 6 individual pieces

Having said this, we want to be clear that when the teacher shows a quantity as a visual image for children to see, it is important to show the "ones" in some easily identifiable pattern, as discussed in Part I of this sequence. The teacher should not show a quantity of ones visually as a group of ones snapped together because often it is not easy to tell in such instances just how many ones there are without counting them by ones. We will say more about this point later on when we talk about possible notation the teacher might use to show children's ways of reasoning.

The wide variety of ways children use the Unifix cubes to act out the story reveal the huge differences in their underlying conceptions of tens and ones. Further, it emphasizes the need to use tasks of this type repeatedly. In the class discussions that accompany each of these tasks there will only be time for some children to explain what they did. After all, there is a limited amount of time in the lesson, but even more, the discussion is not intended as a time when everyone explains his or her way of solving the task. Rather, the discussion is intended as an occasion for learning. It is the teacher's responsibility to orchestrate the discussion so that everyone has an opportunity to advance in their understanding. And the advances children make depend on where they are currently in their conceptual development.

We know that not everyone will get to some predetermined point of understanding after solving and discussing one or two tasks. But the teacher has the opportunity to observe how children use the cubes as they act out the story and she can use that as the basis for deciding which children to call on to explain how they solved a task. And it is her responsibility to ensure that the discussion is meaningful for everyone.

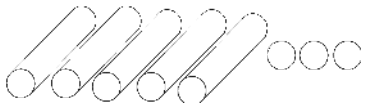
In the case of the second example, the teacher might say, "Amber and Tiffany took away 18 by taking them one at a time, but Destiny thought of 18 as one roll and 8 pieces. So she took away a roll first, that's 10, so now she had to take away 8 pieces." The teacher can then go on to call on someone by name and ask, "Maria, what do you understand about how Destiny

figured this out?" As the discussion progresses the teacher will find out how the class as a whole reasons and also how individual children reason. And as the discussion progresses, the children will progress in their understandings as they attempt to make sense of what others say. As you can see, the discussions of tasks are not intended to check and see if children got the correct answer but are about how children advance in their learning, and about what the teacher learns about their advances.

We have given one example of a "makes more" task and one of a "sells some" task. And we have indicated that the ways children use the Unifix cubes to act out the stories and figure out how many candies there are now differ substantially. Further, we have shown how these differences reveal the wide range of conceptual understandings of tens and ones children have within any class at any given time. Children make advances as they engage in tasks of these types repeatedly and as they participate in class discussions where they listen to and attempt to make sense of other's thinking and reasoning and as they engage in partner work where they work with one another to solve tasks of these same types. (We will discuss partner work later.)

Solutions that emerge as typical and ways to record them
We see several solution approaches that emerge as typical as children use tasks of these types repeatedly. Let's take a look at one more example of each type of task, a makes more task and a sells some task, to see what these typical approaches are.

A "sells some" example:
Mrs. Wright had this many caramel candies.



She sells 15 of them. How many does she have now?

Hope begins by getting out 5 rolls and a group of 3 candies, which she has snapped together. To show that Mrs. Wright sold 15 candies, Hope takes away one roll, the group of 3 pieces she has, and breaks 2 pieces off of another roll. This leaves her with 3 rolls and a group of 8 pieces snapped together, which she easily knew was 38 candies.

Lucas started the same way as Hope. He got out 5 rolls and a group of 3 candies, which he had snapped together. To show that Mrs. Wright sold 15 candies, Lucas took away one roll and broke 5 pieces off of another roll. This left him with 3 rolls, a group of 5 pieces snapped together and the initial group of 3, which he easily recognized this as 38 candies

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Hope's final result

Lucas' final result

In the discussion of this task the teacher will ensure that children make sense of Hope's solution and of Lucas' solution. Questions such as, "Amelia, what do you understand about how Hope figured this out? (Or how Lucas figured this out?)" are useful because such questions get others involved in the conversation. These others describe the solutions in their own words or ask questions if they want further clarification. And that brings even more children into the conversation as still others explain what they do or do not understand.

Another useful question is, "Are Hope's way and Lucas' way of figuring this out the same or different? How are they the same or different?" This question is at a different level because it is asking students to think about each solution as an entity, a thing in and of itself, and compare it to another. For example, in the discussion of this question students might talk about how both Hope and Lucas started off with 5 rolls and 3 pieces. And they will notice that both started by taking away a roll, for the 10 in the 15. (As an aside, children almost always take away the tens first.) It is at that point that the two solutions differ. Hope took away the 5 by using the 3 pieces that were there initially and then 2 more pieces from another roll. Lucas, on the other hand, took all 5 that he needed to give the customer away from one roll, leaving 5 from that roll and the 3 pieces that were there initially. As this part of the discussion progresses the teacher will surely use her Unifix cubes to demonstrate both ways of taking away the 15 for the class to see.

One of the challenges of this type of discussion is that demonstrating how the cubes were taken away is a dynamic process so that as soon as the demonstration is finished, there is no record that shows the difference in the two approaches, except that in this case the final configurations of cubes the two children had differ. As children become familiar with these types of solutions over time, the teacher might want to focus more attention on how the solutions differ. To do so, it is useful to have some way to keep a record of the solutions that the teacher can refer to that help to clarify the difference. The question is how to make a static record of action that is dynamic. We have found that drawings are useful for this purpose.

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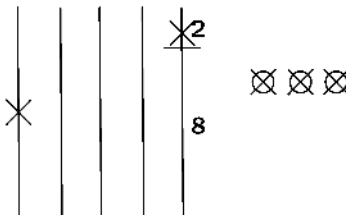
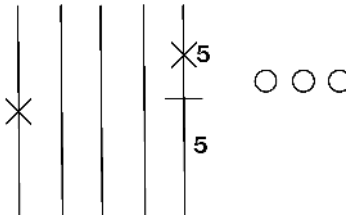
Here we show drawings that the teacher might use, beginning after several days of working with tasks like this.



Record of Hope's solution



Record of Lucas' solution

The teacher will talk through this notation as she writes it. First, she will explain that rather than draw a picture of a roll she will draw a line to represent a roll. She will talk through Hope's solution as she draws 5 lines to represent the 5 rolls and then draws the 3 pieces. She will then draw an x through one roll to indicate that Hope sold that roll to the customer and she will draw an x through each of the three pieces to indicate that she gave those to the customer. Next the teacher will draw a small horizontal line through the roll on the right and write the numerals 2 and 8 next to it. The x next to the 2 indicates that Hope took 2 pieces from that roll to give to the customer. She is left with 3 rolls and 8 pieces.

Likewise, the teacher will talk through Lucas' solution as she draws 5 lines to represent the 5 rolls and then the 3 pieces. For Lucas' solution she shows that he used 5 pieces from a roll to give to the customer. He is left with 3 rolls, 5 from the roll he unpacked, and the 3 original pieces. There are 38 candies left.

It is possible to tell how Hope and Lucas solved this task by looking at the completed drawings. The drawings are records of their solutions approaches. They show how their approaches differ.

As we said earlier, we suggest that the teacher not use this type of notation on the first day of using tasks of this type but begin to use it after several days, after children have had time to develop ways of solving tasks of this type that make sense to them. This type of notation is especially useful when comparing solutions to take note of similarities and differences.

It is our experience that this notation is very useful. Teachers find it easy to use and students quickly understand it as well. The beauty of this notation is that the teacher does not show the 10 individual pieces in the roll that is used to complete the number of pieces given to the customer but instead uses numerals to show the number of pieces from that roll that were sold. In this way this notation not only serves the function of being a record of the solution, it also shifts the focus to thinking quantitatively rather than in terms of individual pieces that would otherwise have to be counted by ones.

Coordinating Units Notes for Video Group 2
E.Yackel

A “makes more” example:
Mrs. Wright has this many butterscotch candies.

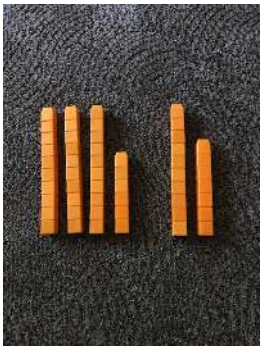
She makes 17 more. How many does she have now?

There are two typical solutions for tasks of this type. We will illustrate them by showing how Samantha and Reginald used their cubes to solve the task.

Samantha got out 36 candies by getting out 3 rolls and 6 pieces, which she left snapped together. Next, she got out 17 as follows. She got out one roll and broke a group of 3 off of another roll to have a group of 7 (snapped together). Now that she had 36 and 17 she combined them like this. She took four of the 7 and snapped them together with the 6 to create another roll. Her final result had 5 rolls and 3 pieces, which she easily knew was 53 candies.

Samantha’s way of using the cubes

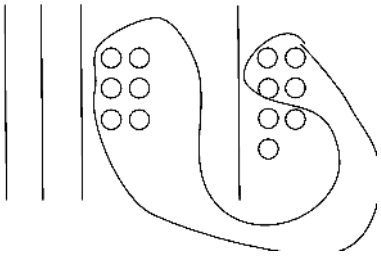
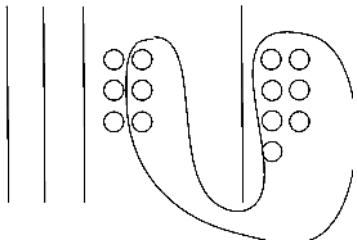
Reginald started the same way Samantha did. He got out 36 candies by getting out 3 rolls and 6 pieces, which he left snapped together. Next, he got out 17 by getting out one roll and breaking a group of 3 off of another roll to have a group of 7 (snapped together). He combined the 36 and 17 like this. He took three of the 6 and combined them together with the 7 to create another roll. His final result had 5 rolls and 3 pieces, which he easily recognized as 53 butterscotch candies.



Reginald's way of using the cubes

As with the "sells some" example we have just discussed, during the class discussion of the "makes more" task the teacher will surely use Unifix cubes to demonstrate the solution approaches Samantha and Reginald. And as we noted with the prior example, the teacher's demonstrations are dynamic. Once completed there is no record to show what happened. In fact, in this case the final configurations of cubes that result from the solution approaches are identical. It is impossible to tell just by looking at the final configuration of cubes these children had that they packed up the candies to create another roll in different ways. Both approaches result in 5 rolls and 3 pieces.

Once again, we suggest that after several days of using tasks like this one, the teacher begins to use notation to record how the task was solved. Here we show notation that teachers have used successfully for this purpose.

Record of Samantha's solution Record of Reginald's solution

These drawings show the initial 3 rolls and 6 pieces and then the roll and 7 pieces that represent how many more candies were made. The loop around the pieces shows how each child thought of combining pieces to pack up into another roll. Samantha used 6 of the original pieces along with 4 of the additional candies that were made while Reginald used all 7 of the additional candies that were made along with 3 of the initial 6 pieces to pack into another roll. In both cases there are now 5 rolls and 3 pieces, for 53 candies.
Notice that in both drawings 4 rolls show up as lines while the fifth roll is shown with the loop around the candies that are packed to make the fifth roll. As an aside, we note that this

approach of drawing a loop around candies to show that they make up a roll is similar to what children did in the circling candies activity in Part I of this sequence.

We want to make one additional comment here. In our discussion of Samantha's and Reginald's solutions we remarked that both children had cubes that represented the pieces in the initial 36 and in the 17 more that were made snapped together rather than broken apart. But here we show them as individual circles. This is important in the notation the teacher shows because children are looking at the teacher's notation from a distance so it is important that they can clearly see how many pieces there are. For this reason it is also important to show those amounts in easily recognizable patterns. Here we show 6 as two columns of 3 and 7 as one column of 4 and another of 3.

We have one additional note of caution about this notation for the "makes more" tasks. It is important to **not** draw an additional line (a roll) to show that another roll was made because then there is confusion about what is represented by the loop and that line. Rather, leave the newly packed roll shown by the looped group of candies. In our experience, teachers that draw an additional line inevitably introduce confusion.

Anticipation Activity and Reasoning Without Unifix Cubes Activity

At the beginning of these notes we listed the instructional activities in this part of the coordinating units instructional sequence as these:

- Reasoning using Unifix cubes
- Anticipation activity
- Reasoning without Unifix cubes
- Imagination bingo
- Partner activities using Unifix cubes

The first four of these activities are conducted in the whole class setting. These four activities follow a progression. Initially students reason by using Unifix cubes as pretend candies. The activities evolve to "reasoning without the cubes" with the anticipation activity intermediary in this progression. Imagination bingo is a culminating activity that is used sparingly, only after most of the students are able to reason entirely mentally, with the support of visual imagery. So far, we have discussed the first activity, "Reasoning using Unifix cubes." Now we want to explain the progression to "Reasoning without Unifix cubes" activity and the role of the anticipation activity in that progression.

As children repeatedly solve tasks using the Unifix cubes to act out the story and to figure out how many candies Mrs. Wright has now, both in the whole class setting and in partner work, they become increasingly efficient in their use of the cubes as pretend candies. Children that initially counted by ones, even though cubes were already packed into tens (rolls) in their supply of cubes, shift to being able to use the sticks of ten as rolls without breaking them apart or counting them by ones. Children such as Joe and Michael, and Amber and Tiffany, shift to using approaches such as Destiny and others used early on. And

children such as Destiny begin to anticipate the results of their actions with the pretend candies, even before they finish acting out the tasks.

At this time it is appropriate for the teacher to shift to using the anticipation activity. In this activity the teacher shows (not flashes) the initial quantity using her cubes as an image for students to see. After students get out this initial quantity using their cubes, the teacher then tells the students the rest of the story—either how many more candies are made or how many are sold. (As an aside we recommend that the teacher write down this quantity for students to see so they can refer back to it if they choose. For example, the teacher might write "makes 34 more" to indicate that 34 more candies are made or she might write "sells 18" to indicate that 18 candies are sold.) Students are then told, "Try to figure out how many Mrs. Wright has now without acting out the rest of the story with your pretend candies. See if you can figure it out just by looking at what you have and thinking about what you **would do** if you were to finish acting it out. How many would you have?" After children have had time to figure out the result, the class engages in a discussion of children's solutions. Children talk about how they would have shown more candies made and how they would pack them up into rolls or about which candies they would have given to the customer and how they know how many would be left. The teacher can use her cubes to demonstrate these ways to the class as the discussion progresses.

The anticipation activity is a transition activity in the sense that students begin to act out the story but interrupt their acting out activity to attempt to figure out the task just by imagining completing the actions with the cubes. We know that students like Destiny and others like her begin to anticipate the results of their actions when tasks are posed in the "reasoning using Unifix cubes" format. The anticipation activity provides them with the needed impetus to advance while at the same time provides those students such as Joe and Michael and Tiffany and Amber the opportunity to complete the tasks using the cubes, if they need to do so. (We often see children such as these gradually shift from actually using the cubes to complete the task to touching the cubes as they reason without acting out the final step of the story.) In this way the anticipation activity provides learning opportunities for children at differing levels of conceptual development of coordinating units of tens and ones to participate meaningfully in the tasks and to advance in their understandings.

After using the anticipation activity for some lessons, the teacher will shift again to the activity we call "reasoning without the cubes." In this activity the teacher poses the task by showing (not flashing) an image of the initial quantity for students to see and then telling the rest of the story, either how many more candies are made or how many are sold. But now students no longer have Unifix cubes available to them to use but are expected to solve the task entirely mentally, but with the support of the visual image that shows the initial quantity.

As teachers watch how students complete the tasks in the whole class setting, they are able to make informed decisions about when to shift from posing the tasks in one form to another. Often children's explanations indicate when they are anticipating the results

without finishing acting out the story. When a number of children appear to be doing so, the teacher can make the shift to the anticipation activity. In fact, teachers often pose one or two tasks where students use the cubes and then pose the immediately following task in the anticipation format. It's easy to do. After she shows the visual image of the initial quantity the teacher can say, "This time, instead of acting out the rest of the story with your pretend candies, just think about how you would do it." Those children that are not yet able to complete the task mentally use their cubes to complete the task.

Similarly, after using the anticipation activity for a number of lessons, the teacher can shift to using the "reasoning without the cubes" activity. Some teachers pose one or two tasks using the anticipation format and then collect the cubes and pose several tasks in the "reasoning without the cubes" format. Even so, there will be a few children that will need to use the cubes to be able to participate meaningfully. It is important that such children be given the cubes, else they cannot participate in a way that makes sense to them. In first grade classes, there may be one or two children that will continue to need the cubes even to the end of this instructional sequence. As every teacher knows, not all children get to the same endpoint in their learning. One principle that undergirds the approach to instruction we advocate is that every child should have a personally meaningful way to participate at all times.

Imagination Bingo

The final activity we list in the progression of whole class activities is imagination bingo. In this activity each child has a bingo card and a supply of chips. The teacher shows (not flashes) a visual image of a quantity of candies that represent the initial amount in a story and leaves that image visible as she tells the rest of the story, either how many more candies are made or how many are sold. Children reason entirely mentally using the visual image shown to figure out how many candies Mrs. Wright has now. They cover that quantity on their bingo cards.

We suggest using this instructional activity only sparingly and primarily as a way to provide a variety in format. In essence, use it as a game. This activity is only appropriate if all but one or two students are able to reason entirely mentally. Those one or two students can be given Unifix cubes to use. However, if the number of students that need Unifix cubes to participate meaningfully is more than one or two, this activity should be omitted.

We remind teachers that the bingo cards should not all be identical else the fun of the game is lost. We also note that the number of possible choices the teacher can use in creating the bingo cards is very large. For this reason, the teacher might choose to limit the numbers used on the bingo cards. And she should plan to use both "makes more" and "sells some" tasks for the bingo game. To make the game move at a reasonable pace, we suggest that there be minimal discussion of the tasks during this game.

Coordinating Units Notes for Video Group 2
E.Yackel

Pragmatic Considerations for Whole Class Activities
There are several pragmatic considerations the teacher should take into account when planning for the whole class activities in this part of the coordinating units instructional sequence.

1. First, always select tasks that require "going over" the decade.
   - Example of task to use: Mrs. Wright has 34 cherry candies. She makes 18 more.
   - Example of task to avoid: Mrs. Wright has 34 cherry candies. She makes 15 more
   - Example of task to use: Mrs. Wright has 46 lemon candies. She sells 18."
   - Example of task to avoid: Mrs. Wright has 46 lemon candies. She sells 14 of them.

   The example "Mrs. Wright has 34 cherry candies. She makes 18 more," is a task that requires going over the decade. Children have to reconceptualize the individual pieces, which exceed 10 in total, as a roll and some pieces. But the example "Mrs. Wright has 34 cherry candies. She makes 15 more," does not require going over the decade. In this task children can consider the tens (rolls) and the ones (pieces) separately and do not have to shift between the two types of units.

   Likewise, a task such as "Mrs. Wright has 46 lemon candies. She sells 18," requires going over the decade. Children have to think about using a roll for some of the pieces that need to be sold. But the task "Mrs. Wright has 46 lemon candies. She sells 14 of them," does not require going over the decade. Children can think about the tens (rolls) and the ones (pieces) separately. They do not have to think about how to use a roll to get some of the pieces they need to sell to the customer.

   The reason for this requirement that tasks always go over the decade is that the goal of this instructional sequence is to foster children's development of the ability to coordinate tens and ones.

2. Second, when the teacher poses the task by showing an image of the initial quantity, that image should always be in easily recognizable form. Here we show several examples.

An easily recognizable form of 34

An easily recognizable form of 29

In Part I of this instructional sequence we explained why the teacher should use a variety of different formats when using the flashing candies activity. Here we are making the opposite recommendation. In the flashing activity the purpose was to create situations where the children encountered more than ten ones (individual pieces of candy). In this way, in order to say how many candies there were they had to think about how to pack all of those individual pieces into another roll. Here, in Part II, our purpose is entirely different. The image the students see of the initial quantity is only the beginning of the task. The reconceptualization that requires shifting between tens and ones (between rolls and individual pieces of candy) comes in as children reason about how they would pack up candies in the "makes more" tasks, and how they would unpack a roll to get additional candies in the "sells some" tasks. For this reason, it is important that children can easily recognize the quantity of candy in the initial amount. In other words, they should not have to work at figuring out what the initial quantity is.

It is for this reason that we recommend that the image the teacher uses to show the initial quantity be easily recognizable. One way to do so, and in fact this is what we suggest, is to use the standard form of showing the rolls on the left and the pieces on the right, with the pieces shown in some pattern children will recognize.

3. A third pragmatic recommendation is that the teacher takes care to always use the language of "makes more" or "sells some" when posing the tasks. Avoid saying that Mrs. Wright **adds** this many candies or **subtracts** this many candies. It is important that children not think, "I'm supposed to add," or "I'm supposed to subtract," "What does that mean?" The entire purpose of using scenarios such as Mrs. Wright Candy Shop is to promote children's development of mathematical reasoning as an outgrowth of what they would do when acting in a realistic setting. The language of adding and subtracting will come in naturally as the sequence progresses.

4. A fourth pragmatic consideration is an obvious one, but nonetheless one the teacher has to attend to. It is this. When selecting tasks make sure that the children's supply of cubes is sufficient for the task.

Partner Activity

So far we have discussed the whole class activities that make up this part of the instructional sequence. Partner work is also an integral part of this sequence. Our intention is that partner work is used throughout and is part of a typical lesson. Here we show a typical lesson in Part II of the coordinating units instructional sequence.

- Whole class activity – The teacher poses tasks that the children solve. A discussion follows each task.
- Partner work – Children work in pairs using Unifix cubes to solve tasks.

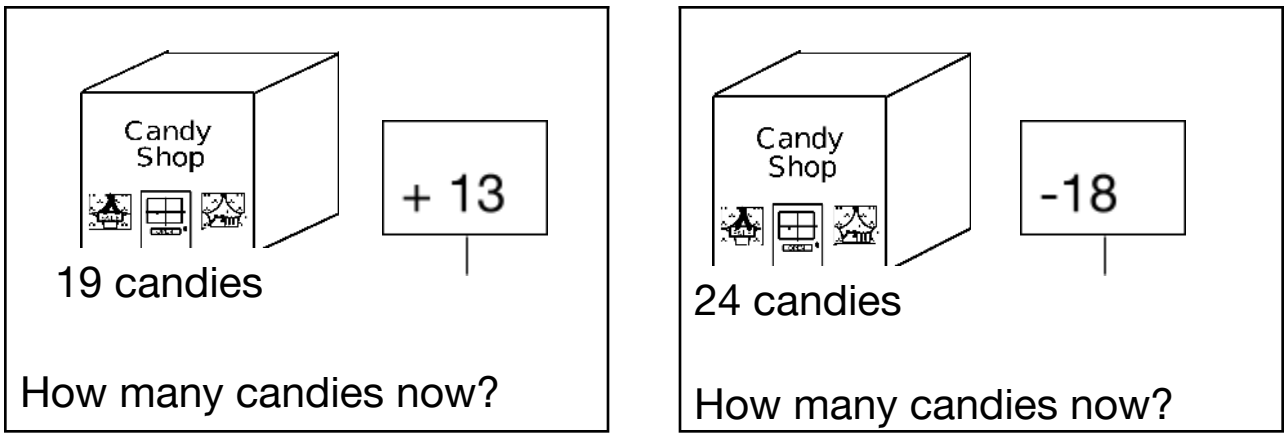
- Subsequent whole class discussion – The class discusses one or two tasks from partner work.

Typically the lesson begins with a whole class activity in which the teacher poses tasks, one at a time, that the children solve and discuss. Thorough discussions are essential and as a result this part of the lesson often takes up to 20 minutes, even if only two or three tasks are used. (As we have already discussed, the nature of the initial whole class activity progresses over time from tasks where children reason by using the cubes to those where children reason without using the cubes.) The whole class work can be followed by 15 to 20 minutes of partner work in which children work with one another to solve tasks using the Unifix cubes as pretend candies. A subsequent discussion of 5 to 10 minutes in which the teacher calls on one or two partner groups to explain one task they worked on concludes the lesson. Allowing for time to get out and put away Unifix cubes and any other materials, this typical lesson will comprise from 45 to 60 minutes time in total. We recommend up to one hour daily for math in the early grades. Our experience is that when the lessons are carefully planned and provide variety in format, such as the lesson we have just described, young children can be easily engaged for up to an hour. Children become very involved when they act out tasks themselves and talk about their thinking and that of others.

One way for the teacher to pose tasks for partner work is to write several on the white board. In this case, the teacher should supply enough tasks so that those children that work quickly will not run out of things to do. (Every child should have some meaningful mathematical activity to engage in at all times during the lesson.) An advantage of this approach is that the teacher can choose tasks based on what has just transpired during the prior whole class activity. If the teacher uses this approach, she has to ensure that children understand that in this classroom persisting on challenging tasks is valued more than completing a large number of them. In other words, children have to understand that they are not expected to complete all of the tasks, but rather work hard to make sense of those they do attempt. The teacher has to also make sure that the subsequent class discussion focuses on a thorough discussion of one or two tasks and not on checking answers or going over all of the tasks posed.

Another approach to posing tasks is to provide each group with a supply of tasks to work on. An approach that we have found to be very effective is to create a set of “task cards” (CS task cards) that can be used for this purpose.

Coordinating Units Notes for Video Group 2
E.Yackel



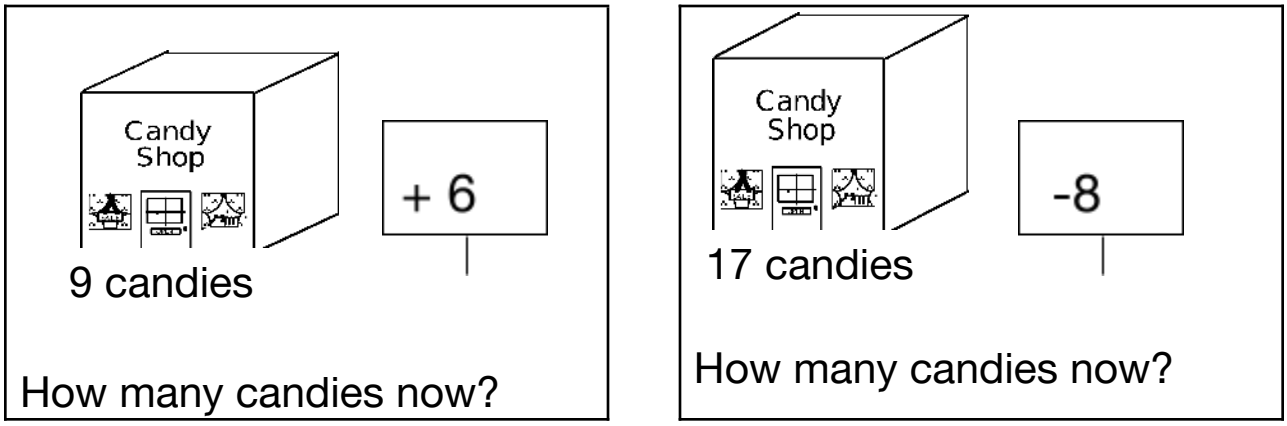
Examples of CS task cards

Here we include one teacher's description of how she used these task cards in her classroom.

> *After posing a few "makes more" and "sells some" tasks in Mrs. Wright's Candy Shop during the whole class activity, I would have my students work with the CS task cards during partner work. These cards have the same kind of tasks that the teacher used during the whole class activity.*
>
> *The way that worked best for my students was to make one set of task cards for each pair of students. I did this by cutting apart the cards and putting each set into baggies. I explained that now they could pretend to work for Mrs. Wright in her Candy Shop and work together to take turns being Mrs. Wright (or a store worker) and the customer by using the story on each card. They loved doing this!*
>
> *While walking around and observing what children did during partner work, I found a variety of ways that they worked with the cards. Some children talked through the scenario together to figure out how many candies Mrs. Wright would have after making more or selling some, while other children took turns being the store worker and the customer. Each pair of students was given a bag of Unifix cubes as pretend candies. I noticed that during the first several days of partner work, the children used the cubes to act out the story, but after several days, I noticed that some children started figuring out some of the tasks without the cubes, even if they had them. The Unifix cubes were always available to anyone who wanted or needed to use them.*
>
> *The same set of task cards was used every time we had partner work. As I walked around the room to see what students were doing and how they were learning, I noticed that students sometimes worked on cards they had previously figured out. To me, this was not an issue because they didn't always solve them in the same way. In fact, the ways they solved the tasks progressed over time indicating their advances in understanding.*

After partner work, we had a subsequent class discussion of 5 to 10 minutes for one or two partner groups to explain a task they worked on. Sometimes pairs of students volunteered to show and explain how they figured out a task. Other times, I called on one or two groups to explain their reasoning because of what I noticed during partner work.

Later, I had the opportunity to work as an inquiry math mentor to collaborate with other teachers new to inquiry math. At one school I worked with, a team of first grade teachers told me that even though their students had worked through the Structuring Numbers sequence, they still needed more time with developing grouping strategies for numbers to 20. Consequently, we developed another set of task cards, "the CS teen cards" for the purpose of giving those students the opportunity to continue developing concepts to 20. That meant that while many students worked with the CS task cards, some students worked with the CS teen cards for partner work during the same lesson. The "CS teens cards" served the dual function of developing concepts to 20 while at the same time fostering development of coordinating units. Both sets of task cards are provided on the website for your use.



Examples of CS teens cards

As you can see from this teacher's remarks, the same types of tasks can be used productively for partner work throughout Part II of the sequence because as this part of the sequence progresses, children make advances in the ways they figure out the tasks.

We want to say a bit more about partner work. For it to be a meaningful learning experience for both partners, it is important that children are paired with someone that is at a similar conceptual level. Here's what I mean. A student who has a better-developed understanding of tens and ones should be paired with another student at a similar level of understanding. A student with a more primitive understanding should be paired with another at a similar level. The reason is that when students who have widely different levels of understanding are paired together, typically the more sophisticated student tells the other what to do and that child often does not understand what the other is talking about.

Coordinating Units Notes for Video Group 2
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Partner work should be a time when each student solves tasks in a way that is personally meaningful to him or her. Teachers should not think of it as an occasion for tutoring. Having said this, it is often not productive to pair the two least sophisticated students together. Rather, pair such students with someone that has already developed a moderate level of understanding.

Another activity that we recommend teachers use for partner work during Part II of the Coordinating Units sequence is the **money trading game**, which is described in detail in the video "Money Activities That Promote Children's Ability to Coordinate Units" and the notes that accompany that video. Teachers we have worked with have found that using this game very often during Part II is very productive in advancing children's ability to coordinate units.

Timeline

In these notes we have described the activities that comprise Part II of the Coordinating Units instructional sequence. These include the activities used in the whole class setting and partner work where the tasks are posed as candy shop stories. Next, we include an overall timeline for the entire instructional sequence followed by a detailed timeline for Part II.

Suggested Timeline for Coordinating Units Instructional Sequence

Week 1 Week 2 Week 3 Week 4 Week 5 Week 6 Week 7 Week 8 Week 9

PART I Becoming efficient with reasoning about coordinating units with a fixed quantity

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PART II Becoming efficient with reasoning about sums and differences by coordinating units

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Activities in Part I of the sequence involve reconceptualizing a quantity by shifting between units of ten and units of one. Part II of the sequence is designed to develop children's becoming efficient with reasoning about **sums and differences** by coordinating units of tens and ones. The activities in Part II involve *changing the quantity* by either increasing or decreasing the initial amount. As the timeline shows we recommend devoting approximately 3 and a half weeks to Part I of the sequence and approximately 5 and a half weeks to the remaining parts of the sequence. Some activities from Part I continue to be used intermittently after Part II begins.

Coordinating Units Notes for Video Group 2
E.Yackel

Now let's take a look at the detailed timeline for Part II.

Suggested Timeline for Part II
of the Coordinating Units of Tens and Ones Instructional Sequence

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9
PART II Becoming efficient with reasoning about sums and differences by coordinating units									
Reasoning using Unifix cubes				----------	----------				
Anticipation activity					----------	----------			
Reasoning without Unifix cubes						----------	----------	----------	
Imagination bingo							----------	----------	----------
Partner activities using Unifix cubes						----------	----------	----------	----------
*Money trading game					----------	----------	----------	----------	----------

* This activity is described in the video "Money Activities That Promote Children's Ability to Coordinate Units."

The first four of these activities are conducted in the whole class setting. They are:

- Reasoning using Unifix cubes
- Anticipation Activity
- Reasoning without Unifix cubes
- Imagination bingo

Each activity is used for several weeks. However, these four activities follow a progression. The overlap in the timeline shows this progression because the nature of the initial whole class activity progresses over time from tasks where children reason by using the cubes to those where they reason without using the cubes.

After several weeks of using the first activity, the teacher shifts to using the second activity when she notices that some of her students are becoming increasingly efficient in their use of the cubes as pretend candies. While transitioning to the second activity, the teacher continues to use the first activity in the same lesson to allow her least sophisticated students the opportunity to use the cubes to enact the entire Candy Shop story. After several lessons of using activities 1 and 2 together in the same lesson, the teacher can shift to using the third activity. She then combines activities 2 and 3 in the same lesson for several more weeks. Some teachers may choose to use the first three activities in the same lesson for a number of lessons.

Eventually the first activity drops out and the teacher uses the second and third activities in the same lesson for a few more weeks. As you can see from the timeline, eventually the

second activity will drop out and the teacher will only use the third activity, although one or two students may still need the cubes to reason with. This gradual shift of moving to the next activity and including the previous activity in the same lesson provides learning opportunities for children at differing levels of conceptual development to participate meaningfully and to advance in their understandings.

Teachers use the fourth activity after using the second and third activities for some time. This activity continues through the end of the sequence as teachers use it sparingly to provide variety in the format.

As the timeline shows, we recommend using the Money Trading Game for partner work during this part of the instructional sequence in addition to partner work in which children solve candy shop tasks using the Unifix cubes. The timeline indicates when each is appropriate.

The Money trading game is described in detail in the video, “Money Activities That Promote Children’s Ability to Coordinate Units.” This game is appropriately used here because it complements Part II of the coordinating units sequence.