Notes for Introduction Video

This video is the first of a series of videos that describe two instructional sequences for early number. These sequences were initially developed in classroom-based research projects and have been revised and refined as they have been used in numerous classrooms.

The sequences are:

- Patterning and Partitioning and
- Structuring Numbers.

They are designed to foster development of a comprehensive and flexible conceptual understanding of numbers to 20, with Patterning and Partitioning specifically designed to develop the conceptual foundation for numbers to 10 and Structuring Numbers designed to develop the conceptual foundation for numbers to 20, with an emphasis on the teen numbers. In this sense, these sequences form the foundation for a well-developed number sense.

The series of videos is organized in the following way. After this initial, introductory video, the video segments are divided according to various instructional activities. The next three videos: Finger Patterns, Dot Patterns, and Single Ten-Frame make up the Patterning cluster. These are followed by a single video that describes three activities designed to promote Partitioning. Next there is a video that describes four Additional Activities for Small Numbers.

The videos that describe the Structuring Numbers instructional sequence follow next. The intention is to keep each video segment to approximately half an hour. The videos can be viewed in isolation from one another. However, they will be most meaningful if viewed in sequence because some will reference earlier material. These videos can be used by individual teachers or in a group setting. In that case, we encourage teachers to pause the videos from time to time to discuss questions and issues as they arise.

Both sequences were originally designed for grade 1. They are intended for the first half of the first grade year. Aspects of these sequences are also appropriate for grades K and 2, when adapted appropriately. The various videos will include timelines that indicate when each activity can be used productively.

These sequences follow an *inquiry approach* to instruction. In contrast to common approaches to mathematics instruction where teachers provide explanations and rules and procedures that students are expected to follow, in the inquiry approach that we advocate, teachers pose problems and tasks that students are expected to solve in ways that make sense to them. The norms that characterize our approach are these.

- Each student is expected to solve tasks in a way that is personally meaningful to him or her.
- Students are also expected to explain their thinking and reasoning to others. Discussion is an integral part of the instructional approach. Students do not simply report their answers to be assessed by the teacher or by classmates. Rather they are

expected to explain their thinking and reasoning. By that we mean that students are expected to clarify their thinking so others can make sense of it.

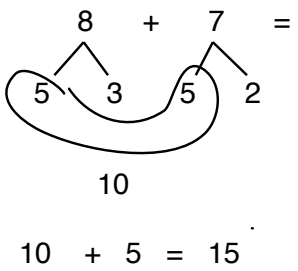
- Students are also expected to listen to and attempt to make sense of others' thinking. The phrase "attempt to make sense" of others' thinking is deliberate and signals that children at differing levels of conceptual understanding might not necessarily have the conceptual basis for understanding what everyone else says. But they should try to do so.
- For class discussions to function effectively it is important that students indicate their agreement or disagreement with what others do and say and feel free to ask questions and offer comments.
- Finally, students are expected to persist on personally challenging problems and to value such persistence over completing a large number of tasks.

Let's think a little more about why we might want to use these instructional sequences, Patterning and Partitioning and Structuring Numbers. We've already said that they provide the foundation for number sense for numbers to 20. I want to elaborate on that a little bit. To do so, let's consider the following. How might young children that do not already know what 8 + 7 is, figure out the answer? When we have asked teachers this question, the most common answer they give is that children will count on, "9, 10, 11, 12, 13, 14, 15." How does the child know when to stop counting? Why does he know to stop when he has said 15, if he doesn't already know the answer before he started? Children have a variety of ways to know when to stop counting. Many put up fingers one at a time as they count on and stop when they have seven fingers up. 9, 10, 11, 12, 13, 14, 15.

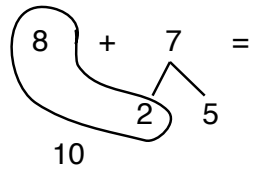
Another answer that teachers often give is that the child might know that 7 + 7 = 14 and so conclude that 8 + 7 has to be 15. Or the child might know that 8 + 8 = 16 so conclude that 8 + 7 has to be 15. These two solutions are called *thinking strategy* solutions.

The term *thinking strategy* is a technical term. It doesn't just mean that the child was thinking. We call a solution a thinking strategy solution when the child uses something he already knows, or has just figured out, to solve the task at hand. These thinking strategy solutions are clearly more efficient than counting solutions.

What if a child reasons like this? I know that 8 is made up of 5 and 3 and 7 is made up of 5 and 2. I put the two 5's together to get 10, the 3 and 2 together to get 5 and the 10 and 5 to get 15.

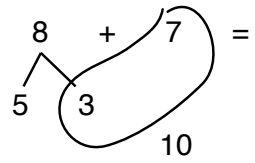


Or like this. I take 2 from the 7 and put it with the 8 to get 10; then I add on the other 5 (left from the 7) to get 15.



10 + 5 = 15

Or like this. I take 3 from the 8 and put it with the 7 to get 10; then I add on the other 5 (left from the 8) to get 15.



10 + 5 = 15

These three solutions are also very efficient and reveal considerable flexibility in working with the numbers involved. These solutions are called *grouping solutions*. In each case the child partitioned the numbers involved into groups that he could easily combine to get other friendly numbers, in this case a 10 and a small number. A quick analysis of these solutions shows that they rely on being able to partition numbers and recombine the parts in efficient ways. In these examples doubles, 5's and 10's have important roles. Invariably teachers agree that they would be delighted if their students could reason in these efficient ways.

So the question is, "How do children develop these strategies?" To answer that question we first ask what it is that children have to know and be able to do to use those strategies. In each of the examples we've given, the child partitioned (or split) one or both of the numbers into parts. In each case, the child then recombined numbers to get a 10. That meant that he had to know the combinations that make 10. Further, in each example, the child went on to add 10 and a small number to get a teen number. Imagine that a child does not yet know that 10 and a small number gives the teen number. Then getting a 10 is of no advantage. Why not get a 9 or an 8 or some other number. The only reason getting a 10 is an advantage is if the child "just knows" that adding a small number to 10 gives the related teen number.

In addition, the child has to be able to use all of this knowledge flexibly. What do I mean? One example we showed was a solution to 8 + 7 where the child partitioned the 7 into 2 and 5, added the 2 to 8 to get 10 and the 10 and 5 to get 15. Why did he partition 7 into 2 and 5 and not 3 and 4 or 1 and 6? To us that's obvious. He wanted to use the 8 to get a 10

and he needed 2 more to do so. But what if the task had been 6 + 7? This time to use a similar approach the child needs a 4 to combine with the 6 to get 10. So this time partitioning 7 into 2 and 5 is not helpful. Instead partitioning 7 into 4 and 3 is useful.

In other words, in addition to knowing how to partition numbers, which combinations make 10, and how adding to 10 yields the teen number, the child has to be able to bring to bear the aspects of all of this knowledge that are relevant for the particular task at hand. Another way to say it is that the child has to develop a comprehensive understanding of the "world of numbers" as a whole, in this case numbers to 20.

That leads us back to our question. How do children develop these strategies? Or, in other words, what does math instruction look like if our goal is that children develop these ways of thinking and reasoning? Our answer to this question is to base instruction on Patterning and Partitioning followed by Structuring Numbers around doubles, 5's and 10's. Patterning and Partitioning focuses on numbers to 10 and Structuring Numbers focuses on numbers to 20, specifically the teen numbers.

These two sequences form the first part of the first grade year. They can also be used productively in the adjacent grades. Kindergarten teachers make use of the Patterning and Partitioning sequence but extend its use over a longer period of time. And second grade teachers often find it useful to use the Structuring Numbers sequence in a curtailed fashion during the early weeks of the second grade year.

As we have already mentioned, in subsequent videos we describe in detail the instructional activities that comprise these two sequences.