Notes for Teacher's Role Video Group
E. Yackel

Notes to Accompany the Video Group:
"Teacher's Role in Inquiry Mathematics Instruction"

Inquiry mathematics instruction is dependent on successful implementation by the teacher. In this two-part video group we highlight two aspects of the teacher's role:

- Instructional planning
- Conducting productive class discussions

While we have included some discussion of these aspects of the teacher's role throughout the three video groups that detail the instructional activities that comprise the Structuring Numbers instructional sequence, in this video group they are our focus.

The first aspect is instructional planning. Here our focus will be on planning lessons that provide all students with opportunities to advance in their mathematical learning. We use specific examples from the Structuring Numbers instructional sequence to show what the teacher takes into account as she plans. In the process we illustrate the principles that undergird the planning process.

To address the second aspect, conducting productive class discussions, we use a brief excerpt from one class discussion to illustrate techniques for orchestrating productive discussions. Throughout our presentation of the episode we explain the purpose of the teacher's actions and the potential effects on the way students participate in the lesson and on their learning. The excerpt is from a lesson in the Structuring Numbers instructional sequence.

Instructional Planning

The focus of the first part of this video group and of these accompanying notes is on the teacher's role in instructional planning. Whereas the teacher's role in conducting productive class discussions, the focus of the second part of this video group is seen in her moment-by-moment decisions and actions in the classroom as the lesson progresses, the teacher's role in planning lessons is seen in her decisions prior to the beginning of each lesson. Here we focus on two specific aspects she considers as she plans a lesson. These are:

- Which instructional activities to use to comprise the lesson.
- Which specific tasks to pose for any given activity.

Principles that Guide Instructional Planning

Before we get into the specific details of the teacher's planning activity, we explain principles that guide her planning. We list them here.

- Every student should have a personally meaningful way to participate in every activity.
- Every student should have an opportunity to advance in his or her learning in every lesson.
- Lessons should provide variety in the format of the activities.

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The first principle is that every student should have a personally meaningful way to participate in every activity. In any classroom children are at differing stages of conceptual development and, with that in mind, lessons are designed so that each child has some meaningful way to participate in each activity. In other words, the teacher does not select activities that only some children are able to participate in. Each child participates in every activity but does so in a way that makes sense to him or her. Some children participate in more sophisticated ways than others but each child participates in one way or another.

And that brings us to the next principle. *Every student should have an opportunity to advance in his or her learning in every lesson*. That means that lessons have to include activities through which all students can advance, including those students that are more conceptually sophisticated. Yet these activities have to also be doable in some meaningful way by everyone in the class. The less sophisticated students will participate in these activities but will use more primitive methods, methods that are appropriate for them. In other words all students participate but they do not all participate in the same ways.

The reason is it possible to plan lessons based on both of these principles simultaneously is that the instructional activities in the inquiry sequences we use are designed to promote *conceptual development*, not teach specific topics or skills. By that we mean that students make advances in their concepts of number and in the ways they reason through engaging in the activities. The goal of an activity is never that students learn "to do" the activity or complete it in a certain way. Rather, it is that all students will advance in their conceptual understanding of number as they engage in the activity. And as they advance, they are able to complete mathematical tasks and solve mathematical problems.

One implication of our approach is that the teacher has to pay close attention to the ways the students in her class reason and solve problems and she has to think through what that means about the current conceptual understandings of the class as a whole and of specific individuals within the class as she makes decisions about which instructional activities to use in each lesson she plans. In other words, the teacher's instructional planning is driven primarily by the current thinking and reasoning of the students in her class, rather than by a preset timeline or outline. This requires that the teacher reflects daily on how the lesson transpired and uses that as input for planning the next day's lesson.

A final principle of lesson planning is a pragmatic one. It is this. It is useful to include activities that provide a variety of formats within a lesson, such as whole class activities and, when appropriate, partner activities or game type formats.

To reiterate, Lessons are designed to:

- Engage all children at their own level of conceptual development.
- Provide opportunities for all children to advance in their learning.

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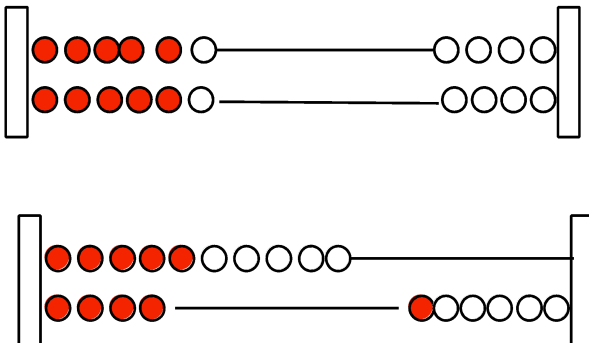
We now illustrate this approach to planning with examples from Parts I and II of the Structuring Numbers sequence.

Selecting Which Activities to Include in the Lesson

First we illustrate how the teacher makes decisions about which activities to include in the lesson.

Example 1. A lesson from Structuring Numbers Part I: *Becoming efficient with "reading" the rack*

Let's think through what the teacher takes into account as she plans this lesson. Her students have been working with the arithmetic rack for several days so have worked through the introductory activities and have already spent several days working on becoming efficient at reading the rack. In fact by now most students in the class have already started to use the color groupings of the beads to figure out the quantity shown. As she reflects on how her students have been engaging in the activities, the teacher considers what they currently understand and what understandings they still need to develop.
She thinks about Maria, for example, who after these few days already capitalizes on the color configurations of the beads. Maria "sees" groups of 5 red beads or 5 white beads, recognizes them as 5 and immediately knows that two groups of 5 beads like this make 10. She counts on by one from 10 to get the total to account for any additional beads beyond those in groups of 5. Maria does not yet know the relationship between 10 and the teen numbers. By that I mean that Maria does not yet "just know" that 10 and 3 are 13 or that 10 and 7 are 17. She counts on by ones from 10 to figure that out. So instructional activities, such as those shown here, that involve reading the rack where the total shown is a teen number that includes two groups of 5 beads of uniform color will be particularly beneficial for Maria's advancement.



The teacher anticipates that in tasks like these Maria will immediately recognize the two groups of 5 beads, that these total to 10, and then will count on to figure out the total number of people on the bus. The teacher also knows from experience that students like Maria develop the relationship between 10 and the teen numbers from repeatedly solving

tasks like these, especially if the teacher ensures that the conversation includes comments such as, "Maria figured out that there were 10 red beads and then she saw 2 white beads and she figured out that makes a total of 12 beads."

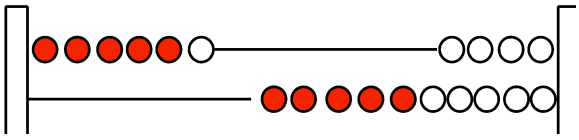
The teacher knows that there are quite a few other children in the class whose understandings are like Maria's. They recognize groups of 5 beads, easily combine two groups of 5's to get 10, but count on beyond 10 to account for other beads. However, the teacher is also aware that one or two students already "just know" the relationship between 10 and the teen numbers. These few students do not count on by ones from 10 to figure out the total. They figure out how many beads there are beyond 10 and then "just know" the total is the relevant teen number.

The teacher also knows that two or three of her students still count all of the beads by ones. Armando is one of them. When asked how many red beads (or how many white beads) are on each rod, Armando will answer, "5" but he cannot yet use that information to solve a task because he has not yet developed the concept of 5 as a composite unit. That means that the only way he can use a 5 is to first "create" it by counting one by one from one to get to five. Armando will not be able to read the rack efficiently until he develops 5 as a composite unit.

Armando will benefit greatly from tasks where he has to show a quantity under 10 on each rod as part of the task because every time he counts such a group he will be at 5 when he has counted all of the red beads. Eventually he will make that realization and will then no longer need to count each bead to get to 5. He will have developed the concept of 5 as a composite unit. That is, 5 is a thing, an entity, in and of itself. So the activity where the teacher tells the students the number of people on the upper deck of the bus and on the lower deck of the bus, the students show that on their racks, and then figure out how many people are on the bus will provide important learning opportunities for Armando.

So with this information in mind, how might the teacher decide which instructional activities to include in the lesson?

She decides to begin with the activity where students have their racks, she tells them how many are on each deck of the bus, the students show that on their racks and then figure out the total. She plans to select tasks where the number on each deck is 5 or more.



Task where first part is to show 6 on the upper deck.

Armando and those students like him will especially benefit because they will have repeated opportunities to encounter groups of 5. This same activity has potential benefits for Maria and those students like her as well because they will encounter the type of tasks

they need to engage in repeatedly. They will easily "see" 10 and then have to figure out the teen number that is the total. The one or two most sophisticated students in the class might no longer learn anything new from this activity. Even so, his or her contributions to the discussion will be useful to everyone.

The teacher decides to also include the rack flash activity in the lesson to ensure that the hi-fliers also have opportunities to advance. In this activity students do not have their racks but "see" an image of the rack that the teacher flashes on the screen. The students' task is to figure out how many people are on the bus from the image flashed. The more sophisticated students will develop mental imagery for quantities to 20 based on the color configurations, images that are 5-referenced, 10-referenced, and doubles-referenced.

The rack flash activity is also beneficial for Maria and those like her because they will quickly develop mental imagery for groups of 5 and 10 and will be able to use that mental imagery to go beyond 10 to get to the teen number. And in keeping with the principle that every student should have a personally meaningful way to participate in every activity, the teacher will make sure that Armando and those few students like him have a rack to use during the rack flash activity. These students might not be able to complete the entire task before the discussion begins but they will be able to move beads on their racks to show the same configuration the teacher has flashed. And during the discussion these students will be able to use their racks to continue to solve the task and to make sense of the accompanying discussion.

Both of these instructional activities are whole class activities. To provide variety in the format the teacher decides to include a partner activity. In partner work children have an opportunity to be more physically active and necessarily are all fully engaged. In this case the teacher's choice is to use the partner version of the first whole class activity she has planned. In this activity the partners take turns being the teacher, posing tasks to each other. This activity will provide additional opportunities for all of the students to advance their learning because they get to choose the difficulty level of the tasks they pose to each other and because with partner work children tend to pose many tasks to each other within a short amount of time. The teacher decides to use this activity second in the lesson. So now her overall plan looks like this.

1. Whole class activity: The teacher tells students how many people are on each deck of the bus. Children show that on their racks and then figure out how many people are on the bus.
2. Partner activity: This is a partner version of the above whole class activity.
3. Whole class activity: Rack flash (Students do not have their racks, although one or two might need a rack.)

In keeping with our recommendation, this teacher typically devotes close to an hour to each math lesson. So she plans on having students engage in each of the three activities for a little more than 15 minutes. That allows for a few minutes of transition time between activities, including getting out and putting away the racks, as needed.

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Teachers typically use a plan such as the one we have shown in example 1 for several days, possibly a week or more, before adjusting it by omitting one or more of the activities in the lesson and including one or more activities that are more conceptually challenging. In this way the teacher organizes lessons to gradually progress through the sequence. In essence the sequence evolves. For additional discussions of this issue see the concluding slides of each of the video groups Structuring Numbers 1, Structuring Numbers 2, and Structuring Numbers 3

We want to emphasize again that by providing increasing conceptual challenges within a lesson the teacher ensures that every child has an opportunity to advance in each lesson. At the same time, because each child participates in a way that is personally meaningful, some children will use solution methods that are less sophisticated than others. For example, in some cases the teacher will make sure that children that need racks will have them to use while other children will not have access to a rack for that same activity.

Example 2. A lesson from Structuring Numbers Part II: *Becoming efficient with adding and subtracting using grouping strategies prompted by the rack*

Instructional activities planned for this lesson include:

1. Whole class activity: Reasoning by moving the beads (Students have their racks.)
2. Whole class activity: Anticipation activity (Students have their racks. They show the initial quantity on their racks but instead of showing the change on their racks they think about how they would do so and what the result would be.)

Teachers that use the Structuring Numbers instructional sequence successfully typically make gradual shifts in the activities they use. The lesson shown in this second example illustrates this point. This example is from Structuring Numbers Part II: becoming efficient with adding and subtracting using grouping strategies prompted by the rack. The teacher might use this lesson after students have been working with the "reasoning by moving the beads" activity for several lessons. She knows from experience that after just a few days of using the "reasoning by moving the beads" activity many children move the beads to show the number of people that get on or off the bus in very efficient ways, ways that result in configurations that can be easily read by using the color configurations of the beads. Their activity appears to be purposeful. This suggests that such children know *before* they move the beads which ones they will use to complete the task.

At this time it is appropriate for the teacher to introduce the anticipation task. It is easy to do. After posing several tasks where children move the beads to complete the task, the teacher can begin to pose the next task in the same way, but after students have shown the initial quantity on their racks she can say, "This time instead of moving the beads to show what happens, just think about how you *would* move them and figure out what the result will be."

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What the anticipation activity does is call explicit attention to the ways children think about the second part of the task. Which beads would they move to show that more people got on the bus (or some got off the bus)? In the discussion of this task, the teacher asks the children to talk about which beads they *would have moved* to complete the task and *why* they would have moved those beads in addition to how they figured out the result. Doing so requires that children articulate their reasoning using language that others can understand. That means that they have to be able to verbally express any mental imagery that might be guiding their actions. The result is that those children that are still touching and moving the beads to complete the task become explicitly aware of how choices of which beads to move lead to various grouping strategies. In this way these children gradually also develop the ability to anticipate the results without actually completing the task with the rack. In other words, the anticipation activity provides opportunities for all of the students to advance. The more sophisticated children advance as they make their thinking and reasoning explicit, while the less sophisticated students advance by listening to and attempting to make sense of what others say while also having the opportunity to complete the tasks by moving the beads, if necessary.

Most teachers would plan a third activity for this lesson that involves a different format, such as a pairs activity or a game. In our experience teachers often include an activity of that type from an earlier portion of this same instructional sequence, an activity that they know still provides considerable learning opportunities for their students.

Teachers typically use this overall plan for several lessons, possibly up to two weeks, before adjusting it by deleting the first activity and adding the activity "reasoning without moving the beads." We refer you to the suggested timeline in the Structuring Numbers 2 video group.

Selecting Specific Tasks for the Activities to be Used in the Lesson

A second aspect of the teacher's advance planning is to select the specific tasks she intends to pose for an activity she will use within the lesson. Here again the teacher is purposeful in her choices as she thinks through the potential learning opportunities these choices provide. Again we will give two examples to illustrate considerations that the teacher takes into account in deciding which tasks to plan. The first example is taken from the first part of the Structuring Numbers sequence, *becoming efficient at reading the rack*. The second example is taken from the second part of the Structuring Numbers sequence, *becoming efficient with adding and subtracting using grouping strategies prompted by the rack*.

Example 1. Activity: Figure out how many people are on the bus when the configuration is specified. Specific tasks posed in this order:

1. 6 people on the upper deck and 5 people on the lower deck.
2. 6 people on the upper deck and 6 people on the lower deck.
3. 6 people on the upper deck and 7 people on the lower deck.
   (Or 5 people on the upper deck and 7 people on the lower deck.)

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In the activity in this example, the teacher tells students how many people are on the upper deck of the bus and how many are on the lower deck of the bus. Students show each of those amounts on their racks and then figure out how many people are on the bus in total. Recall that this is not an addition task. Rather, students are expected to use the beads on their racks to figure out the total by capitalizing on the color configurations of the beads. For this reason it is important that the teacher choose numbers that result in configurations that can be "read," that is, figured out, efficiently. In the following discussion we illustrate the teacher's thinking process as she chooses which tasks to pose.

The first task the teacher plans is

- 6 people on the upper deck and 5 on the lower deck.

She plans this as the first task because she expects that children like Maria will figure out that there are 11 people on the bus by combining the two groups of 5 red beads, the 5 red beads on the top and the 5 red beads on the bottom, to get 10 and then know that the total is 11 because there is also one white bead. The teacher expects that Maria's explanation will be useful to Armando and those children like him that did not use that approach but instead counted the beads. They will take note of that explanation, attempt to make sense of it, and some of them will be able to use that way of reasoning if presented with a similar task. Therefore she plans for that possibility by her choice of the second task.

The second task she plans to pose is:

- 6 people on the upper deck and 6 people on the lower deck.

The teacher knows from experience that more children will figure out this task by combining the two groups of red beads to get 10 and then know that the total is 12 because there are 2 white beads. Some children will just know that 10 and 2 are 12 and others, like Maria will count 10, 11 12. Nevertheless, she also knows that some children, like Armando, will continue to count by ones because they have not yet developed a concept of 5 as a composite unit.

She also expects that at least one or two children will solve this second task another way by reasoning, "it's just one more, one more than 11. There's one more white bead." These children are not using groupings of beads in efficient ways to solve this second problem but are simply noticing that there is one more bead than last time. These students are capitalizing on the fact that they have already figured out the initial task so all they need to do is add one more.

The teacher plans to use

- 6 people on the upper deck and 7 people on the lower deck

as the third task because it provides the same opportunities as the second one did. Some children will capitalize on the two groups of 5 red beads to get 10 and continue on with the white beads to get 13. Others will relate it to the previous task, noticing that there is just one more bead, so the answer has to be one more than 12. It has to be 13.

The teacher plans another task that she might use as an alternative for the third task.

- 5 people on the upper deck and 7 people on the lower deck

Her reasoning for this choice is that this task again creates an opportunity for students to group the red beads to get 10 and then get 12 by combining the 2 white beads. It also creates an opportunity for children to reason, "there's one less bead on the top and one more bead on the bottom so it has to be the same as last time." This is another type of relationship between tasks that students might capitalize on because of what they notice about the beads.

The teacher is aware that this choice of tasks creates opportunities for children to use color configurations of beads to solve the tasks or to solve them by relating them to a prior task. These three tasks may take up all the time she wants to devote to this activity in this lesson because the class discussions take a considerable amount of time. However, she plans a fourth task in the event that there is time for it.

She plans two alternatives for a fourth task, should she have time for it. The first alternative is:

- 5 people on the upper deck and 6 people on the lower deck.

She expects that some children will solve this task by relating it to one of the prior tasks. For example, some might notice that it uses the same quantities as the first task but the quantities are switched around on the rods. And she expects that other children will again combine the two groups of red beads to get 10 and go from there. She will use this task if time permits because it provides opportunities for both of these solution methods.

The second alternative the teacher has planned as a fourth task is:

- 10 people on the upper deck and 5 on the lower deck.

She chose this task because the quantities involved in this task are not easily related to those in the prior tasks so students will not be able to solve it by relating it to the prior tasks. She plans to use this task in the event that most students are solving the tasks by relating them to one another and few are using the color configurations of the beads to solve them. She will use this task if she decides to bring the focus back to using strategies based on the color configurations of the beads. She chooses the numbers for this task deliberately. By using 10 on the upper deck and 5 on the lower deck, she makes it possible for students to combine all of the red beads (5 from each rod) to get 10, just as some of them did for the first three tasks or to get 10 from combining the 5 red beads on top with the 5 white beads on top and then go on from there. So in using this task she can bring the focus back to using strategies based on the color configurations of the beads while yet providing a new learning opportunity because of the 10 on the top rod.

In the three video groups Structuring Numbers 1, Structuring Numbers 2 and Structuring Numbers 3 and the notes that accompany each video group we have devoted some discussion to task selection and in some cases have talked about the benefits of sequencing tasks. In the example shown here, we have provided extensive detail about the rationale for the teacher's choices. It is important that teachers plan in advance which tasks they will pose in the lesson to maximize learning opportunities for their students. At the same time, during the lesson itself, teachers have to make judicious decisions about when to deviate

from their planned sequence of tasks to capitalize on "teachable moments" that arise spontaneously.

Example 2. The second example of a task sequence a teacher might plan is taken from the second part of the Structuring Numbers sequence, *becoming efficient at adding and subtracting using grouping strategies prompted by the rack*. The task sequence we describe here can be used effectively with any one of the activities:

- Reasoning by moving the beads, where children use their racks to solve the tasks.
- Anticipation activity, where students show the initial quantity on their racks but instead of moving the beads on their racks to complete the task, think about which beads they would move and use that to figure out the result.
- Reasoning without moving the beads, where students see the initial quantity on the image the teacher shows and then complete the task entirely mentally.
- Imagination bingo.

The first task the teacher has planned is:

- 5 people on the bus, 7 more get on

She expects that many students, like Maria, will use color groupings of beads to solve this task, most likely combining the 5 red beads on the top rod with the 5 red beads on the bottom rod to get 10 and then figure out the result is 12 because there are also 2 white beads. By now students like Armando may have progressed to use this approach as well.

The second task the teacher has planned is:

- 5 people on the bus, 8 more get on

She chose this for the second task to provide an opportunity for students to use a strategy based on color groupings of beads, similar to that used for the first task, or to reason that this is just one more than the first task because there is one more bead.

The third task the teacher has planned is:

- 13 people on the bus, 8 get off

The teacher expects that some students will relate this task to the previous task, reasoning that it is just the "opposite." In this way this third task contributes to students' developing understanding of the reversibility of addition and subtraction. The teacher knows that other students will use a strategy based on groups of beads, either taking 3 from the bottom rod (i.e., emptying the bottom) and then taking away the other 5 from the 10 beads on top to get down to 5, or taking all 8 from the 10 (beads on the top rod) to leave 2 beads on top and 3 on the bottom for a total of 5 people left on the bus. Whether they actually move the beads or just think about moving the beads will depend on which activity is involved.

The fourth task the teacher has planned is:

- 13 people on the bus, 9 get off

The teacher selected this task to again provide an opportunity for students to use groupings of beads to solve the task or to relate it to the prior task. She knows from past experience that students who do not relate tasks initially often do so after they hear others reason in

that way. The teacher knows that this sequence of tasks creates opportunities for children to solve each task after the first one by reasoning based on groups of beads or by relating tasks to one another based on changes they see in the beads from one task to the next. So this sequence makes it possible for children to advance along two different lines of reasoning

We want to reiterate here how this sequence of tasks provides learning opportunities for all students at a variety of levels to advance in their reasoning. We also want to emphasize that the goal is not that everyone "sees" the same relationships. This is another example of how a sequence allows for differentiation.

When the teacher wants the focus of the discussion to revolve entirely around using color configurations of beads to solve the tasks she will not use this sequence but will instead choose tasks that have no apparent relationship to one another.

Task sequences and thinking strategies. We want to make one additional important point about the sequences of tasks shown in the two examples presented here. Throughout our discussion of these examples we have noted that the teacher's selections of tasks has made it possible for students to use the color groupings of the beads to solve the tasks or to solve subsequent tasks by relating them to prior tasks simply by visually noticing some change in the number of beads, such as that a task might have one more bead than the prior task. At first glance, this second approach to figuring out the tasks might seem to be cause for concern because it appears that the children are relying only on a visual cue. But the point we want to make is that solutions of this type are the genesis of another type of reasoning that we want to foster, thinking strategy reasoning.

We say that someone uses a thinking strategy if they use something they already know or have just figured out to solve the task at hand. Typically we use the label thinking strategy to apply to those situations where the task is posed numerically. For example, if the child's task is to solve 6 + 7 posed purely numerically or in a numerical setting without visual support material and the child says, "I know that 6 + 6 = 12 so 6 + 7 has to be 13. It's just one more," we say the child solved the task using a thinking strategy. His solution was based on reasoning about the quantities involved.

A natural question is, "How does the teacher promote children's development of thinking strategies?" One way to do so is for her to use task sequences such as those shown in these examples. The fact that visual material is present in the posing of the tasks makes it likely that someone will notice the relationship between the tasks because of the visual cues. And through the discussion, the relationships between the quantities involved become explicit. As the structuring numbers sequence progresses and the visual support of the rack drops out, reasoning by relating tasks to one another becomes independent of the visual support material.

In other words, in the same way that grouping strategies emerge from children's use of the arithmetic rack as they become independent of the visual support of the rack, thinking

strategies emerge as well if the teacher plans sequences such as those shown here where visual material is initially available but later drops out. Another way to say it is that the task sequences shown in these examples provide opportunities for children to develop the basis for both grouping strategies and thinking strategies.

Orchestrating Productive Whole Class Discussions

In the second part of this video group and these accompanying notes we illustrate techniques for orchestrating productive class discussions, including comments and questions the teacher poses, and the reasons for her choices. Whereas the teacher's planning activity is carried out in advance, before the lesson begins, her actions in conducting the class discussions take place during the lesson. She continually makes moment by moment decisions-in-action throughout the class discussions. For this reason, it is essential that the teacher develop well thought out techniques for orchestrating the discussions that are based on sound underlying rationale.

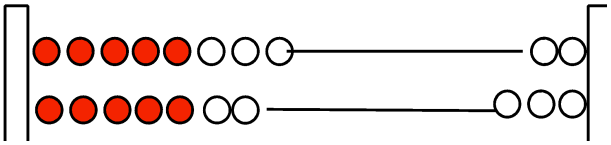
These are some issues for consideration:

- Who does the teacher call on and why?
- How does the teacher engage everyone in the class discussion?
- What kinds of questions or comments might the teacher use to find out if students are making sense of others' thinking and reasoning?

In the process of discussing these issues we demonstrate both *that* and *how* the discussion make it possible for everyone to move forward.

We have selected a brief excerpt from a whole class discussion and analyze it segment by segment to show how the teacher addresses these issues by being deliberate in her remarks and questions. The entire excerpt comprised about 5 to 7 minutes of the lesson.

The example we use comes from the first part of the structuring numbers sequence, *becoming efficient at reading the rack*. The specific activity the class is engaged in comes early in the sequence. Each child has an arithmetic rack for this activity. The teacher projects the configuration shown here on the screen.



Teacher: Make yours look like mine. Try to figure out how many people are on the bus without counting. See if you can use the colors of the beads to help you figure out how many people are on the bus. (Pause.)

Teacher: Please raise your hand if you need more time to figure out the number of people on the bus.

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She says, "Make your look like mine." Her additional remarks, "Try to figure out how many people are on the bus without counting. See if you can use the colors of the beads to help you figure out how many people are on the bus," are deliberate. They emphasize the goal of this activity, which is to use the color groupings of the beads to figure it out.

The teacher allows time for students to complete the task. But before calling on anyone she says, "Please raise your hand if you need more time to figure out the number of people on the bus." In this way, the teacher lets the children know that she will give them sufficient time to solve the problem. This approach differs from traditional instruction where the teacher typically calls on students to respond within 5 to 10 seconds of posing a question or problem. Here the teacher does not pose questions and tasks that students are expected to already know the answers to. Rather, students are expected to figure out the answers to questions or to solve the tasks in ways that make sense to them personally. That means they need sufficient time to do so.

The dialogue continues as the teacher asks, "How many people are on the bus?"

Teacher: How many people are on the bus?
Roberto: 15
Sammy: 14
Tanesha: 15
Marco: 15
Paris: 15
Carla: 15

Notice that the teacher calls on a number of students to give their answers but does not yet ask them to explain how they figured it out. Her rationale for doing so is that she wants to get some idea of the range of answers. After all, she has no idea what the students figured out. Further, doing so gives students the opportunity to think about others' answers as well. Now the teacher has to decide which student to call on to explain how they figured it out. In this case, the teacher calls on Sammy.

Teacher: Sammy, how did you figure out 14?
Sammy: I saw 7 on top and I counted 7, 8, 9, 10, 11, 12, 13, 14.
Teacher: Does anyone have a question or comment for Sammy?

There are several points of significance in this brief exchange. Why did the teacher call on Sammy? After all, he had the wrong answer. But that is precisely the reason she called on him. Notice that the teacher does not engage Sammy in a conversation to get him to correct his error. Rather, the teacher opens up the discussion to get everyone involved by asking, "Does anyone have a question or comment for Sammy?" In the approach to instruction we advocate, the teacher's approach is the same when the student has a wrong answer as it is when no mistake is made.

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Let's see what happened. The teacher continues to engage students in the class discussion by asking them questions.

> Teacher: Who figured it out like Sammy? Did anyone have a different way of figuring this out?

These questions shift the discussion away from the children's answers to their solution methods. Each child now has to think about the way he or she solved the problem and compare it to Sammy's way. The dialogue continues.

> Leon: I counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
> Teacher: Thank you, Leon. Leon counted to figure this out. Did anyone use the color groupings of the beads to figure this out?

We want to emphasize two points regarding this brief exchange. First, in offering his solution Leon was presumably aware that it was different from Sammy's. He counted all of the beads by ones whereas Sammy counted on from 8, the 8 beads on the top rod. Even though Leon used a primitive solution he was able to notice that it differed from Sammy's. Second, the teacher is aware that some of the children still need to count by ones so she thanked Leon. However she did not spend more time discussing his solution method because the goal of this activity is to develop efficient ways based on the color groupings of the beads to figure it out. Her question, "Did anyone use the color grouping of the beads to figure this out?" shifts the discussion to this emphasis.

The dialogue continues.

> Paris: I know that 10 and 5 make 15.
> Teacher: Does anyone have a question or comment for Paris?

Notice that the teacher's question here is exactly the same as the question she asked earlier after Sammy gave his solution. Her purpose is to get others involved in the discussion. The most obvious question to ask Paris is "Where did the 10 and the 5 come from?" It is significant that the teacher refrained from asking this question herself. She did so because she knows that other children in the class are surely wondering the same thing. So instead of asking that question, she invited others to ask questions or make comments so they could make sense of Paris's thinking. And this is exactly what happened. Roberto asked exactly that question.

> Roberto: Where did you get the 10 and 5?
> Paris: I put the 5 red beads on top with the 5 red beads on the bottom to make 10. I put the 3 white beads on top with the 2 white beads on the bottom to make 5. I know that 10 and 5 make 15.

And Paris answered it by explaining, "I put the 5 red beads on top with the 5 red beads on the bottom to make 10. I put the 3 white beads on top with the 2 white beads on the bottom to make 5." And she reiterated, "I know that 10 and 5 make 15."

So the teacher's approach was effective because it resulted in the students independently asking questions and making comments to get more information in order to understand Paris's thinking. The benefit goes well beyond getting the class involved in the discussion. It also helps to establish that the teacher is not the sole mathematical arbiter in this classroom. All of the students can ask questions, call for clarification, or make remarks. In fact, they are not only allowed to, but are expected to do so.

Now that Paris explained how she got the 10 and the 5, the teacher wants to know if others in the class understood her reasoning. To find out what other students understood and to encourage them to try and make sense of how Paris figured it out, the teacher might ask the these questions.

Teacher: Can you help us understand how Paris figured this out?
Teacher: What do you understand about what she said?"

The teacher might ask these questions of the whole class or she might specifically call on someone by name. In our example, the teacher decided to call on a specific student, Zane.

Teacher: Zane, can you help us understand how Paris figured this out? What do you understand about what she said?"
Zane: Paris made 10 with the 5 red beads on the top and the 5 red beads on the bottom. She made 5 with the 3 white beads on top and the 2 white beads on the bottom. She put the 10 red beads together with the 5 white beads to make 15**.**

We know from Zane's explanation that he understood how Paris figured out the task, because he explained her thinking in his own words.

The dialogue continues as the teacher asks students to compare Paris's way of thinking to Sammy's way.

Teacher: Is Paris's way the same as or different from Sammy's way?
Bianca: Different.
Teacher: How are they different?
Bianca: Sammy saw 8 on top and counted 8, 9, 10, 11, 12, 13, 14, 15. Paris used the two fives to get ten and put the ten with the 5 white beads to get 15.
Teacher: Does anyone have any comments or questions?
Tanesha: I had a different way of figuring it out. I put the two 5 reds together to get a 10. Then I counted the white beads: 11, 12, 13, 14, 15.

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In this brief excerpt we see that Bianca took up the challenge the teacher posed and explained how the solutions are different. The teacher's next question is less specific. She asks if anyone has any comments or questions. In doing so, the teacher opens the discussion more broadly. Children are now free to talk about anything, not only about comparing Paris' and Sammy's solutions. Tanesha uses this opening to talk about her own way of thinking. Many times students will listen to others' ways of reasoning and think about how theirs is the same or different, then bring that into the class discussion as Tanesha did here, even though the discussion is purportedly about comparing Paris' and Sammy's ways of solving the task.

Now, the teacher has a decision to make. Does she want to go on to another task, or spend more time with this one? In this case, the teacher decided to wrap up the discussion of this task and go on to the next task. We remarked earlier that the entire episode discussed here comprised only 5 to 7 minutes of the entire lesson. Yet, the detailed analysis we have presented shows how the teacher used a variety of techniques to ensure that the class discussion was productive. To emphasize this point, we include the entire dialogue below as a continuous segment.

Teacher: Make yours look like mine. Try to figure out how many people are on the bus without counting. See if you can use the colors of the beads to help you figure out how many people are on the bus. (Pause.)
Teacher: Please raise your hand if you need more time to figure out the number of people on the bus.
Teacher: How many people are on the bus?
Roberto: 15
Sammy: 14
Tanesha: 15
Marco: 15
Paris: 15
Carla: 15
Teacher: Sammy, how did you figure out 14?
Sammy: I saw 7 on top and I counted 7, 8, 9, 10, 11, 12, 13, 14.
Teacher: Does anyone have a question or comment for Sammy?
Teacher: Who figured it out like Sammy? Did anyone have a different way of figuring this out?
Leon: I counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
Teacher: Thank you, Leon. Leon counted to figure this out. Did anyone use the color groupings of the beads to figure this out?
Paris: I know that 10 and 5 make 15.
Teacher: Does anyone have a question or comment for Paris?
Roberto: Where did you get the 10 and 5?
Paris: I put the 5 red beads on top with the 5 red beads on the bottom to make 10. I put the 3 white beads on top with the 2 white beads on the bottom to make 5. I know that 10 and 5 make 15.

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Teacher: Can you help us understand how Paris figured this out?
Teacher: What do you understand about what she said?
Teacher: Zane, can you help us understand how Paris figured this out? What do you understand about what she said?"
Zane: Paris made 10 with the 5 red beads on the top and the 5 red beads on the bottom. She made 5 with the 3 white beads on top and the 2 white beads on the bottom. She put the 10 red beads together with the 5 white beads to make 15.
Teacher: Is Paris's way the same as or different from Sammy's way?
Bianca: Different.
Teacher: How are they different?
Bianca: Sammy saw 8 on top and counted 8, 9, 10, 11, 12, 13, 14, 15. Paris used the two fives to get ten and put the ten with the 5 white beads to get 15.
Teacher: Does anyone have any comments or questions?
Tanesha: I had a different way of figuring it out. I put the two 5 reds together to get a 10. Then, I counted the white beads: 11, 12, 13, 14, 15.

We want to make one additional important point about the example we have presented here. It is this. The teacher did not call on all of the children to report how they figured out the task, nor did she even call on many to do so. The purpose of the class discussion is not to gather solutions from the children. Rather, it is to focus on specific conceptual aspects that have the potential to advance learning. It is the teacher's responsibility to ensure that the discussions do just that. She is responsible to make sure that the discussions "go somewhere."

The issues illustrated by the analysis of the dialogue are:

- How the teacher decides whom to call on.
- Ways the teacher engages everyone in the discussion.
- How the teacher encourages students to try and understand and make sense of other students' thinking, and how she knows what they understand.

We have not pointed to any one excerpt from the dialogue that demonstrates how the class discussion makes it possible for everyone to advance in his or her learning. The teacher techniques that contribute to that are those that help to establish that students are expected to listen to each other and attempt to make sense of each other's thinking. Here's what I mean. As students listen to each other and attempt to make sense of what others say, they inevitably interpret what they hear in terms of what they currently understand. And when children hear something that is within or just at the boundaries of their current understandings, they are often able to make sense of it and incorporate that thinking into their own. In this way, through participating in the discussion, even as listeners, students are able to advance to new levels of understanding. Of course, they also have the opportunity to ask questions they may have.

This points to an important difference between the approach to instruction that we advocate and traditional math instruction. In the latter, the teacher is typically the person that brings new ways of thinking and reasoning into the discussion that she then expects the students to make sense of and replicate. But here it is the students that typically bring ways of thinking and reasoning into the discussion that other students may not have thought of on their own. And as the teacher works to ensure that students clarify their thinking for others she is ensuring that the discussion is advancing everyone's learning. It is in this way that the teacher also makes sure that mathematical ideas, concepts, and ways of reasoning that are on her own agenda become part of the discussion. After all, she can exercise control over which emphases become the focus of the discussion.

Summary Remarks

In this video group and these accompanying notes we have focused attention on the importance of the teacher's role in inquiry math instruction, using the Structuring Numbers Instructional Sequence as the basis for our discussion.

In the first part of the video group and notes we have discussed the nature of the teacher's planning activity, with an emphasis on how the teacher uses the ways her students engage with the activities and reason to solve tasks as the basis for her planning. In the process we have demonstrated how the teacher takes into consideration the important principles that:

- Each child should have a personally meaningful way to participate in each instructional activity and
- Every child should have an opportunity to advance in his or her learning in every lesson.

In the second part of the video group and notes we have presented an example of a class discussion to illustrate the teacher's role in orchestrating effective class discussions, emphasizing the relevance of the teacher's remarks and questions and how they influence the way the discussion evolves. In the process, we have also demonstrated how children's learning advances through participating in the class discussion.

Additional remarks about the teacher's role can also be found in three video groups that describe the details of the Structuring Numbers instructional sequence, Structuring Numbers 1, Structuring Numbers 2, and Structuring Numbers 3 where such information is interspersed throughout.