Notes for Finger Patterns Video

Finger Patterns begin the Patterning and Partitioning instructional sequence. The sequence is designed to foster children's early development of foundational conceptions of number. Finger Patterns are used throughout the sequence even as other instructional activities are introduced. In this sense, this video is an introduction to the entire Patterning and Partitioning sequence. For this reason, we devote considerable attention to important details about the fundamentals of children's conceptual development of number.

The instruction we describe is based on classroom-based research. It is intended for whole class use and is heavily dependent on the nature of the accompanying classroom discourse. In this video we:

- Describe the instructional activities for finger patterns.
- Explain various ways children might engage in the activities and what these ways indicate about their current number concepts.
- Explicate learning opportunities the activities afford.
- Discuss issues related to classroom discourse.

The approach to instruction we advocate is based on the instructional principles that each child engages in an activity in a way that makes sense to him and that each progresses in his conceptual development to reach a more advanced stage. Such an approach acknowledges that at any given time children in a class are at different stages of conceptual development and will advance to different stages of understanding.

The implication is that the teacher conducts instruction with a keen understanding of these differences and takes mental note of the advances individual children make. Further, the teacher understands that, as in any instruction, it is important to keep pushing forward. This type of instruction is not based on mastery with everyone engaging in a same predetermined way. As we have noted, children engage in an activity in a way that is commensurate with their current understandings. We do not have as a goal that children engage in an activity in a certain way. In this regard we agree with Piaget when he said that students should only say and do what they believe and not something they think they are supposed to do or say.

Each activity has two interrelated parts. In the first part children use their fingers to complete a task posed by the teacher, such as, “Show 7 on your fingers.” In the second part, the class discusses the just completed task in order to promote children’s reflection on the task. As we clarify in this discussion of the finger pattern activities, it is the reflection engendered by the discussion that results in children making advances in their conceptual development. In this way, the activities go well *beyond* promoting development of finger patterns for numbers to 10.

Initial activity

The teacher asks students to use their fingers to "show" a specified quantity, for example, "Show 7 fingers" and then initiates a discussion using questions such as:

- How many fingers do you have up on each hand?
- Is there a different way to show it? Who has a different way to show it?
- Is your way the same as or different from _____'s way? How is it different?
- What do you notice about ___'s way?
- Is ___'s way is the same as or different from ___'s way? Why or why not?

What do children do*?* To "show 7,"

- Some children will count from one putting up one finger with each number word they utter (vocally or sub-vocally).
- Some children might hold up several fingers at once and know how many that is without counting, then put up additional fingers one by one as they count on until they get to 7.
- Gradually most children will hold up all the fingers on one hand at once and know without counting that they have five. At this point, those children have developed a notion of five as unit—a single entity that is made up of five individual units. These children no longer have to "make 5" each time they need it by counting one by one from one.

While it is important for the teacher to take note of these differences in the ways children put up their fingers (one at a time or groups of fingers all at once) these differences are not the subject of conversation, as can be seen from the suggested questions. The focus of the conversation is on the *results* children have, not on how they achieved those results. This focus is deliberate. It requires that children take a retrospective look at their results (and the results of others) and talk about these results.

Here's what I mean. At the initial stages of using this activity children often are unable to tell how many fingers they have up on each hand without going back to count them. For example, children who put up their fingers one by one as they utter the number word sequence most likely need to "create the number from one" each time. They know to stop putting up fingers when the last number word they utter is the number they are to show. For example, a child might put up one finger at a time while uttering (out loud or to himself) the words, "one, two, three, four, five, six, seven." Many children at this stage are not able to tell how many they have on each hand without going back to count each hand from one.

Let's look specifically at a child, Sam, that proceeds in just that way. He put up one finger with each number word until he got to 7. "1, 2, 3, 4, 5, 6, 7." When Sam was asked how many he had on each hand he had to go back and recount the fingers on each hand to figure out that he had 5 fingers on one hand and 2 on the other.

What might the teacher do in this situation and what is the potential effect of that action? The teacher might use her own fingers to reiterate and elaborate Sam's way. She might

hold up her own fingers for everyone to see and count just as Sam did, "1, 2, 3, 4, 5. 6, 7. There's the 5, the whole first hand, and there's 6, 7. That's 2 more, 1, 2."

What's the effect of the teacher doing this? As the teacher says, "6, 7. That's 2 more. 1, 2," she is emphasizing that the fingers that represented 6, 7 can also be thought of as representing 1, 2. In this way the 6, 7 becomes the same as 1, 2. This simple teacher action is the beginning of partitioning 7 into 5 and 2. Or another way of saying it is that we see the genesis of a finger pattern for 7 as 5 on one hand and 2 on the other. We hasten to emphasize that a single exchange of this type does not, in and of itself, result in everyone in the class now just knowing that 7 can be thought of as 5 and 2. Further, just knowing that is not the goal of the activity.

In fact, the finger pattern activity does not have as a single goal that all children will have well-developed finger patterns for numbers to 10. There are multiple goals, including:

- That children move beyond having to "create a number from 1" by counting one by one.
- That children develop five as a composite and that it becomes firmly established.
- That children develop 5-referenced and 10-referenced relationships for numbers to 10.

This multiplicity of goals is essential to effective use of finger pattern activities. Children at any grade are at a variety of different levels of sophistication in their concept of number. Some children have well-developed concepts of five as a composite unit while others in the same class will need to create a number from 1 each time by counting one by one.

Going back to the task of showing 7 fingers, we note that while some children may have done so by putting up 5 fingers on one hand and 2 on the other, other children may have shown 7 by using 3 on one hand and 4 on the other. Maria is a child that did just that. She showed 7 by using 3 on one hand and 4 on the other. What might the teacher do in this case? In this case, the teacher asks, "How many fingers are on each hand?" and Maria responds with, "three and four."

At the earliest stages of using finger pattern activities with the class, the students are just learning that they are to be involved in the class discussions. So at this stage, the teacher usually has to initiate such discussions. One way to do so is to ask questions of the class to get them involved. In this case the teacher might now ask the class, "Maria says she has 3 on this hand and 4 on that hand. Do you agree? How do you know? How can you be sure?" This last question will likely generate various responses from students, including counting the fingers by one because at this stage it is the explanation that goes to the most basic level. Everyone will understand that explanation.

This brings up another important matter for teachers to be aware of. The issue is what it means to explain or justify something. How much needs to be said? In this example we've said that the most basic justification is counting by ones. In other words, counting by one is something that will make sense to everyone in the class and hence is important so our least sophisticated students can understand. At the same time one of our goals is that students who count one by one advance to no longer needing to do so. For this reason, the teacher

has to understand that over time explanations also need to advance so that they do not continue to include the most basic details. Further, it is essential to not send the message to children that have more sophisticated understandings that their explanations should always be at the most basic or primitive levels. Rather, a child's explanation should be at a level commensurate with his understanding.

Now let's go back to our example where Maria showed 7 by using 4 fingers on one hand and 3 on the other and knew how many she had on each hand without going back to count them. Rather than asking students if they agree with Maria, the teacher might instead choose to comment, "Maria just knew that she had 4 fingers on this hand and 3 fingers on that hand. She didn't have to count them. She just knew that."

An apparently simple remark such as this by the teacher has huge implications because it signals to students that their explanations should be at, and not below, their own level. Surely the teacher will go on to make sure that the discussion does include counting by ones if there are students in the class that still need to do so. But students will not come to believe that their own explanations are "supposed to" include these basics.

The reason I include these comments is because we have experienced classrooms where well-intentioned teachers that have worked hard to ensure that class discussions are comprehensible to everyone have unwittingly left the impression that every explanation should go back to the most basic details every time. In these cases, we have seen more sophisticated students abandon explanations of their ways of reasoning and instead always report a primitive way of thinking and reasoning. Another way of saying it is that the finger pattern activity, like all other instructional activities, will progress in the way children engage and talk about it. Finger patterns will eventually drop out altogether as children's number concepts advance.

Classroom conversations are always dependent on what children do and what they offer in the discussion. Nevertheless, the teacher exercises considerable control in that she chooses the questions she asks and/or chooses students to call on. In the case of the above example, with Maria, another alternative is for the teacher to ask:

- Who has the same way as Maria?
- Does anyone have a different way?
- Is 3 and 4 the same number as 2 and 5?

Each of these shifts the discussion from each child's own way of showing the number to a comparison of ways—comparing 5 and 2 to 4 and 3 or comparing each child's individual way to another way. These apparently simple questions about differences elevate the activity to yet another level.

I want to elaborate a bit more on the point of *elevating the discussion*. When we ask children what is the same or what is different, they are put in the position of considering two solutions to the task at the same time and comparing them. In other words, each solution is now taken as a "thing," an entity that can be thought of holistically, and

compared to another. Children's mental activity in this comparison task is different than it is in the original task of holding up the specified number of fingers.

Why is 2 and 5 the same number as 3 and 4? Some children will create each combination and take note of differences in the fingers they used. Others may not be yet able to go beyond figuring out for themselves that each way does show 7. Some children may remark that if you have 5 on one hand and 2 on the other you can take down one finger from the hand with 5 and put up one more on the hand with 2 to end up with 4 and 3. If the child uses fingers to demonstrate that reasoning, more children will be able to make sense of it than if it is just said verbally.

Such a remark shows a nascent, in-action form of a thinking strategy, one that we call the compensation thinking strategy. We hasten to add that we are under no illusion that a child who makes such a remark already has a conceptual understanding of the compensation thinking strategy. Nor are we recommending that the teacher discuss the compensation thinking strategy at this time.

Other important principles, such as that 5 and 2 have the same total as 2 and 5, can emerge as children switch the order in which they hold their hands. It is significant to note how apparently simple actions and remarks foreshadow important aspects of conceptual development that will emerge over time, including initial conceptions of various ways to split, or partition numbers.

Simple Variations

Simple variations of the initial finger pattern activity of holding up a specified number of fingers include the following:

- Show it a different way.
- Show it using two hands.

We have already talked about the ways children engage and the learning opportunities these variations afford.

More Complex Variations

Here are two more complex variations, illustrated by these examples.

- Show 4. Put up 2 more. How many do you have now?
- Show 9. Take down 3. How many do you have now?

These tasks begin to develop the concepts of putting up more or taking away. Notice that the language here is not the mathematical language of adding and subtracting but relates specifically to what the children do with their fingers—put up some more or take some down.

Let's look specifically at the first of these tasks:

Show 4, put up 2 more. How many do you have now?

What do children do? Some children will still be at the stage of counting fingers one by one as they put them up, 1, 2, 3, 4 and 1, 2 and then count the number of fingers they have up at the end, 1, 2, 3, 4, 5, 6. Other children may by now have a finger pattern for some numbers. Such children may show 4 at once then count two more, 1, 2. Some may now count all of their fingers by ones to figure out they have six fingers up. Others may count on saying, 5, 6 as they point to the two additional fingers. Notice that such children can recite numbers in the number word sequence without beginning at one. Still others may just recognize they have 6.

These variations in ways of engaging in the tasks reveal that children that have not yet developed finger patterns for any of the numbers and have not developed concepts of composite unit for any of the small numbers can nevertheless participate meaningfully. The tasks may not yet be about adding more or taking away for them but may still be about ways to show a specified number and ways to figure out how many fingers are up. In this way even though the tasks as posed appear to progress in complexity and in the types of action and reflection they elicit, and therefore in the types of learning opportunities they provide, children that are not yet able to participate at more sophisticated levels have learning opportunities at levels appropriate for them.

Teachers have found that through repeated use of the finger pattern activities, children make significant advances in the ways they engage with the tasks, indicating their advancing conceptions of number. No formal assessments are needed because the teacher can easily see what children do.

We can increase the complexity of the tasks even more by posing tasks such as:

- Show 4. How many more do you need to have 7?
- Show 7. How many do you need to take down to have 5?

What do children do? After showing the initial four, many children continue by counting 5, 6, 7 while putting up one more finger with each number word they utter. As a result the child now has 7 fingers up. If the child showed 4 with one hand and used fingers of the second hand while counting 5, 6, 7, he may easily recognize that the three fingers on the second hand are those needed to get to 7. By now most children will recognize these fingers as 3 without counting them by one. However, a child that put up 4 fingers on one hand and continued by first filling up the first hand before moving to the second while uttering 5, 6, 7 will end up with one full hand and 2 fingers on the other. In this case the challenge for the child is to distinguish which fingers he used to make up the additional ones beyond the initial 4. This challenge will be beyond the child that still needs to make a number by counting from 1. But even this child will listen to and attempt to understand what others are saying as they explain that the fingers put up while uttering 5, 6, 7 represent 3 (1, 2, 3). In other words, this child has important learning opportunities at his own level.

These tasks are building the foundation for missing addend and missing subtrahend problems. As such, they are more demanding cognitively than the previous two tasks and therefore are posed later. Nevertheless, it is important to continue to move forward with

posing more complex tasks in order to provide opportunities for all of the children to advance, including the more sophisticated children. Further, since children participate in the tasks in personally meaningful ways, those children that are less sophisticated continue to have learning opportunities at their own level and hence can advance in their understandings as well. This illustrates an important point. The teacher does not need to provide differentiated instruction. The children do the differentiation themselves by participating at their own level.

Tasks that Promote Reversibility

Tasks of the type exemplified by:

- Show 8. Take down 3. How many do you have now? Now put 3 more up. How many now?

are designed to foster the conceptual understanding that addition and subtraction are inverse operations (often referred to as the *reversibility of addition and subtraction*).

What do children do? Children that by now have well developed finger patterns will put up 8 all at once. Some will do it in two steps first holding up 5 fingers on one hand and then immediately thereafter holding up 3 fingers on the other hand. Such children will fold down 3 fingers all at once and just recognize the remaining configuration as 5. Children will vary in what they do next. Some will put up 3 fingers all at once while others may count 1, 2, 3 as they put up 3 more fingers. Most children will now just recognize the resulting configuration as 8.

The task is more complex for a child that showed 8 as 4 and 4. He is more likely to count 1, 2, 3 as he folds down 3 fingers and see the remaining configuration as 4 and 1 more for a total of 5. This child is likely to use the same 3 fingers he folded down to put back 3 more.

The least sophisticated child might not have been able to complete the entire task. Nevertheless, even this child will be engaged in the initial part of the task, show 8. In this way, this type of task provides continued opportunities for the less sophisticated students.

As with the task types discussed previously, we are under no illusion that children develop understandings of reversibility through using this type of task only once or twice. Repeated use of these task types is essential to such development. Through repetition children have the opportunity to reflect on the totality of their actions rather than on each separate part of the task. The conversation that accompanies these tasks can be the catalyst to promote such reflection.

These remarks highlight the importance of using several tasks of any specific type one after another within the same lesson. For example, in the illustration above, a child may remark after figuring out that the final result is 8, “Hey, that’s the same number we started with.” The teacher can follow up with, “What do you mean?” In this way a discussion can be initiated that has as its focus the relationship between adding an amount and then subtracting the same amount, or vice versa. Perhaps no one has anything to say at this point, but even if someone does, we would not expect every student in the class to make

sense of all that is said. However, if the next task the teacher poses is of the same type, students that did not make the relevant observation on their own on the first task might do so now. They might observe, "We ended up with the same number that we started with this time too." These students are on the verge of developing the relationship between addition and subtraction for themselves. Consequently, posing yet a third task of the same type may be crucial for them.

To explain the rationale for this type of task we have used the term reversibility. We want to be clear that we do not intend that the teacher use this terminology with the students. In fact, we have mentioned earlier that we do not advocate talking about adding and subtracting when using finger patterns. Rather we talk about putting up more fingers and taking some down. Our choice is deliberate. The goal with finger patterns is conceptual development of number. Children's concepts grow out of their actions and reflections on those actions.

Tasks That Promote Combinations of Ten

A final task type we want to discuss is exemplified by

- Show 6. How many more do you need to have 10?

While this task type appears to be similar to one already discussed, we list it separately because the key element of this task is that the total is 10. The ability to come up with combinations that make 10 in any specific situation is foundational to having a comprehensive, flexible world of number, for numbers to 10. Further, any children that have not yet developed 10 as a composite unit, that is, a unit that is itself comprised of ten individual units of one, can do so through this task type.

Once children have well-developed finger patterns that they use spontaneously to solve problems it is time to discontinue using the finger pattern activities as an explicit part of your lessons, typically several weeks in grade 1. That does not mean that we ban finger pattern use. Rather, it is important to recognize their importance in children's explanations. Children frequently use finger patterns when explaining their ways of reasoning, even if there is no evidence that they actually used finger patterns to solve the problem. We have seen a number of instances when a child's use of finger patterns in an explanation appears to be his way of providing what he sees as essential detail for others in the class or of attempting to clarify his thinking in response to requests for additional clarification. Earlier we remarked that, at the beginning of finger pattern use, counting is the explanation that goes to the most basic level. As finger patterns become well established for the class, taken as a whole, *they* become a basic level of explanation. Counting drops out as no longer needed as most basic. The amount of detail the class requires as sufficient for an acceptable explanation has advanced. This shift is evidence that the finger pattern activities have advanced the learning of the class.

Summary

To summarize, in this video we have discussed various finger pattern activities. They are listed here.

- The initial finger patterns activities.
  Show this many fingers.
- Several variations.
  Show it a different way.
  Show it using two hands.
- More complex variations.
  Show _____. Put up _____ more. How many now?
  Show _____. Take down _____. How many now?
  Show _____. How many more do you need to have _____?
  Show _____. How many do you need to take down to have _____?
- A task designed to promote reversibility.
  Show _____. Put up _____more. How many now? Take down _____ (same number as before). How many now?
- A task designed to promote development of combinations of 10.
  Show _____. How many more to have 10?

In presenting these activities we have attempted to show how *they are at once simple and complex*. The goal of the finger patterns activities is conceptual development. Each activity is accessible in some way to every child, no matter his or her level of mathematical sophistication. And each activity has the potential to promote conceptual advancement.

Timeline

Finger pattern activities are productive for a number of weeks and, as the timeline shows, can be used productively along with other activities in the Patterning and Partitioning sequence in first grade. Many grade K teachers use finger pattern activities effectively for many more weeks at the beginning of the school year. In the above discussion we have explained how children at different levels of sophistication participate in the different task types.

Teachers we have worked with have been most successful when they intermingle task types within a lesson as they notice progress the children make and to accommodate children's differing levels of sophistication. Thus finger patterns activities are appropriate for a longer time in grade K than grade 1 and can be used in grade 2 if needed there.

SUGGESTED TIMELINE FOR FIRST 6 WEEKS OF GRADE 1

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
PATTERNING						
Finger Patterns	----------	----------	----------	----------	----------	----------
Dot Patterns		----------	----------			
Single Ten-Frame			----------	----------	----------	----------
PARTITIONING						
Monkeys in the Trees					--- --- ---	
Double Decker Bus						-- --
All the ways						-- --
ADDITIONAL ACTIVITIES						
Hidden Objects			--- --- --- --- ---	--- --- --- --- ---		
Single Bus				--- --- --- --- ---	--- --- --- --- ---	--- --- ---
Doubles		--- --- --- --- ---	--- --- --- ---			
Money					--- --- --- ---	--- --- --- ---