

Structuring Numbers Notes for Video Group 3

E. Yackel

Notes to Accompany Video Group 3: Structuring Numbers Part III

Part III: Becoming Proficient With Using Grouping Strategies to Solve Additive Tasks Without the Rack

Part III of the SN instructional sequence is designed to further develop children's proficiency with using grouping strategies to solve additive tasks independent of the support of the AR.

Instructional Activities

1. Additive tasks without the rack
 - Posed in a context (story)
 - Posed purely numerically
2. Focus on written numerical notation
 - "Read my mind" activity

The primary instructional activities in this part of the sequence are additive tasks posed in a story setting or posed purely numerically that students are encouraged to solve using the grouping strategies they developed through their work with the AR. As we will explain, the teacher's use of notation to record children's solution methods is an important aspect of the class discussions of these tasks. In addition, we will describe a task that focuses even more specifically on written numerical notation.

Additive Tasks Without the Rack

Story contexts. The teacher will pose these tasks in the whole class setting, posing them one at a time and engaging the class in a discussion of their solution methods. We encourage teachers to use story contexts that they know will be meaningful to their students. Contexts that teachers we have worked with have used productively are:

Cookie Jar

Bus

Purse

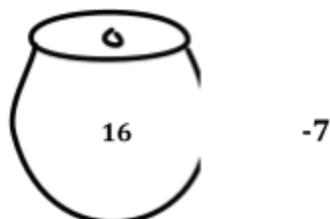
Piggy Bank

We remind teachers to be judicious in selecting numbers in keeping with the purpose of the structuring numbers sequence, that is, a focus on numbers to 20, with an emphasis on going over 10. Number choices such as $7 + 8$ or $15 - 9$ are productive for this purpose but not number choices such as $13 + 4$ or $16 - 5$. Let's look at several examples of tasks of this type.

Example 1. There were 16 cookies in the cookie jar. Amy and her friends ate 7 cookies. How many cookies are in the cookie jar now?

The teacher poses this task by stating the problem aloud while drawing the picture and writing the numbers involved on the picture for students to see.

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After students have had time to figure out the task, the teacher engages the class in a discussion of their solution methods. Remember this is a whole class activity. Children do not have paper and pencil or any other materials to use.

What do children do? We would expect that students will use a variety of grouping and thinking strategies because these tasks are used only after a number of students have developed a conceptual basis for using grouping strategies based on the previous AR activities in Part II of the sequence. Here are some solution methods we can expect.

Samantha: 9. First I took away 6 to get down to 10. Then I took 1 away from 10 and I got 9.

Maria: I thought of 16 as 10 and 6. I took all 7 away from the 10. That leaves 3. I put that 3 together with the 6 to get 9.

Jorge: I know that 16 take away 6 is 10 so 16 take away 7 has to be 9.

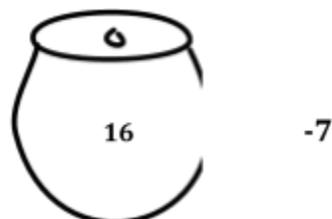
Armando: 9. I counted down. 15, 14, 13, 12, 11, 10, 9.

Samantha's and Maria's solutions are grouping strategy solutions. Jorge's solution is a thinking strategy solution. And Armando's gave a counting solution. We include Armando's solution to clarify that even at this stage in the sequence there might be one or two children that have not yet developed grouping or thinking strategies.

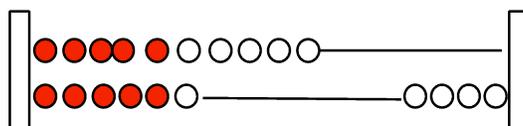
In conducting the class discussion of the task the teacher will follow the inquiry approach we have described extensively in other notes and videos. She will ensure that other students are involved in the discussion, that they ask questions if they need further explanation or clarification, and that they compare and contrast solutions. In these tasks it is also important that the teacher ensure that students give the rationale for their choices. For example, here Samantha explained WHY she first took away 6. She did it to get down to 10. In the notes that accompany Part II of this sequence we have provided extensive discussions of the rationale for students reasoning. Rather than repeat that emphasis here, we will focus on written notation the teacher might use during the discussion to record the solution methods. A written record of the solution methods is important since otherwise students have to rely entirely on what they hear and remember as the discussion progresses.

Samantha's solution. 9. First I took away 6 to get down to 10. Then I took 1 away from 10 and I got 9.

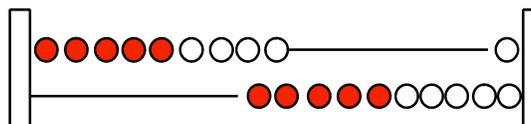
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This solution method grows out of tasks on the AR where 16 is shown as 10 beads on the top and 6 beads on the bottom.



and where the 6 beads on the bottom are moved to the right and 1 bead on the top is moved to the right



We show the rack here only as a reminder of the origins of this way of reasoning but want to be clear that the rack is NOT used in posing this task and students do not have racks to use to solve the task. Furthermore, the AR is no longer part of the discussion as we can see from Samantha's remarks.

The teacher can easily use number sentences to record her thinking.

$$\begin{array}{r} 16 - 6 = 10 \\ 10 - 1 = 9 \end{array}$$

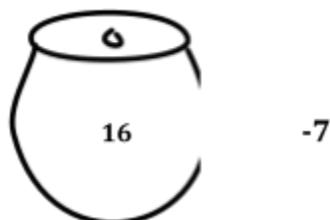
The teacher can go on to point to the 6 and the 1 in the number sentences, or even draw a loop around them, as she remarks, "There's the 7 that Samantha took away. First she took away 6 and then she took away 1 more." Alternatively, the teacher can ask the class, "Can you see the 7 that Samantha took away? Where can you see the 7?" In this way, the teacher is calling specific attention to the numerical notation and specifically inviting students to relate this notation to the verbal explanation Samantha gave.

As a footnote we remark that in the earliest stages of using context problems like this, the teacher might choose to actually show a rack and show the movement of the beads as a further clarification of Samantha's reasoning for those few students that are not yet able to reason entirely mentally without the support of visual materials. However we want to be clear that the teacher does not show the AR as she poses the task. It only comes in if needed for clarification during the discussion.

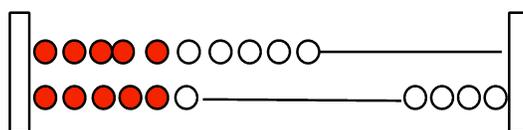
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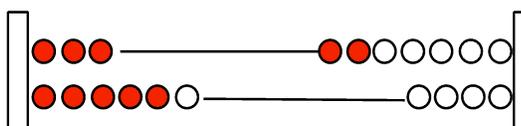
Maria's solution. I thought of 16 as 10 and 6. I took all 7 away from the 10. That leaves 3. I put that 3 together with the 6 to get 9.



Maria's solution method grows out of tasks on the AR where 16 is shown as 10 beads on the top and 6 beads on the bottom



and where all 7 beads were moved to the right on the top.



As was the case with Samantha's solution, here the AR was no longer part of the discussion. But the teacher has previously introduced the class to numerical notation for the rack solution. That notation is appropriate here as well.

$$\begin{array}{r}
 16 - 7 = \\
 \begin{array}{r}
 10 \quad 6 \\
 -7 \quad 6 \\
 \hline
 3 \quad 9
 \end{array}
 \end{array}$$

The notation shows that Maria thought of 16 as 10 and 6 and that she took all 7 away from the 10.

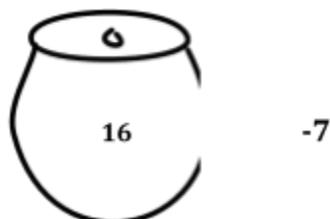
The discussion should not only include elaboration of Maria's method, including inviting students to explain what they understand about it and to ask clarifying questions, but should also include specific discussion of the notation and how it relates to Maria's reasoning. In other words, what we are advocating is that the discussion goes well beyond the students' reasoning to also emphasize the intended meaning of the notation used.

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And again we remark that in the earliest stages of using context problems, the teacher might choose to actually show a rack and show the movement of the beads as a further clarification during the discussion for those few students that are not yet able to reason entirely mentally without the support of visual materials. But showing the AR is not part of posing the task. As use of these tasks continues, the teacher will be able to discern when to discontinue showing the rack even for clarification during the discussion.

Jorge's solution. I know that 16 take away 6 is 10 so 16 take away 7 has to be 9.



Jorge's solution method is not a grouping solution. It is a thinking strategy solution. At first glance, it appears to be almost the same as Samantha's solution. But there is an important difference. While Samantha actually thought about taking 7 away from 16, "first I take away 6 and then I take away 1 more," Jorge was reasoning by relating the task posed to a number fact that he just knows. He is arguing that because 16 take away 6 is 10, 16 take away 7 *has to be* one less than 10. He is basing his reasoning on how the numbers in the two tasks are related to each other. His is a general way of reasoning that is not dependent on the specific numbers involved. For example, someone might argue, "I know that 12 take away 6 is 6 so 12 take away 7 has to be 5." When someone uses something they already know or have just figured out to solve the task at hand, we say they used a thinking strategy solution.

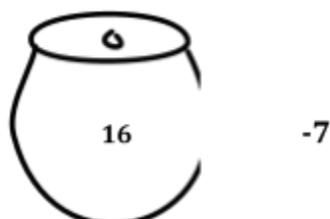
In this case we recommend that the teacher use the notation shown here, simply writing the two number sentences but possibly also writing the word "so" to clarify how Jorge reasoned.

$$16 - 6 = 10$$

$$\text{So } 16 - 7 = 9$$

This solution does not relate specifically to the AR so it is not appropriate for the teacher to show the AR as part of the elaboration of this solution.

Armando's solution. 9. I counted down. 15, 14, 13, 12, 11, 10, 9.



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Even though most children will have developed grouping solutions as they worked with the activities in Part II, there may be one or two students in the class that have not yet done so. Armando is a case in point. He used a counting solution. Our recommendation is that in cases like this the teacher acknowledge the solution but not devote time to it or attempt to notate it in any way. After all, the notation we have been discussing serves an important function. And that function is to link numerical notation to grouping (or thinking) strategies. In this way the notation continues to reinforce the overall goal of the sequence. And, as we have explained in an earlier video, the notation provides a record of the child's thinking and reasoning.

Many teachers keep the notation for the various solutions children give for a task visible through the entire discussion. In this way, children have an opportunity to take note of similarities and differences. To illustrate what we mean, this is what the white board or screen would look like after the discussion that includes the four solution methods we have given here. Notice that there is no notation for Armando's solution since it was a counting solution.

16 - 7

SAMANT
HA

$$16 - 6 = 10$$

$$10 - 1 = 9$$

MARI
A

$$16 - 7 =$$

10 6 9

-7

3

JOR
GE

$$16 - 6 = 10$$

So $16 - 7 = 9$

Now that all of the solutions have been discussed, the teacher can go back to the notation and ask the class how the solutions were alike and how they were different. How does the notation help to show the similarities and differences? The class is now being asked to compare, not only the reasoning these children used, but also the notation the teacher used to record that reasoning. In this way the notation itself becomes an object of the discussion. And, as it does, the children's solution methods become increasingly linked to quantity as represented by numerals, rather than as represented by the visual images of the beads on

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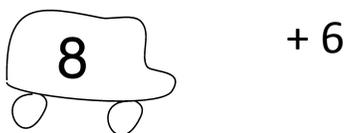
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the AR. What we are trying to say is that this culminating discussion is more than just a summing up of the task. It has the potential to contribute to children's continuing advancement to reasoning entirely quantitatively without the support of the AR.

We can also imagine a situation where some child, Max, replied, "I got 9," and said no more. Certainly as children become increasingly proficient at solving additive tasks without any visual support, they often say less and less by way of explanation. They might simply say, as Max did, "I got 9," and not feel the need to provide any elaboration. They might assume that others in the class require no additional explanation. After all, a goal is that children become proficient with number relationships to 20. And as they develop these relationships, over time many of them become essentially automatic. It becomes counterproductive if the teacher always insists that such children provide details that they might no longer need for themselves. However in the early stages of using tasks such as this, typically children are using some form of reasoning to figure out the task. So it is reasonable for the teacher to ensure that the child explain how he did so. And, in any case, if someone in the class calls for clarification, then it is incumbent on the student to provide additional explanation to clarify how he or she figured it out.

Example 2. There were 8 people on the bus. 6 more people got on the bus. How many people are on the bus now?

Bus



The teacher poses this task in the whole class setting as describe previously. Children solve the task entirely mentally.

What do children do? Here are some solutions we would expect children to offer.

Louis: I got 14. I broke the 6 into 2 and 4, put the 2 with the 8 to get 10, and added the 10 and the 4 to get 14.

Danielle: I put 4 of the 8 with the 6 to get 10 and then I added on the other 4 from the 8 to the 10 to get 14

Christian: I took 5 out of the 8 and 5 out of the 6. I added the two 5's to get 10. I added the 3 left from the 8 with the 1 left from the 6 to get 4. 10 and 4 make 14.

Ben: I counted on from 8. 9, 10, 11, 12, 13, 14.

Joe: I know that 6 and 6 equal 12 so 8 and 6 are 2 more. I got 14.

Sue: I know that 8 and 8 are 16. So 8 and 6 are 14.

The first three children used grouping strategies. In each case the child partitioned the numbers and recombined them in efficient ways. Ben's solution is a counting solution. The

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last two solutions are thinking strategy solutions. Of course it is possible that only some of these solutions might be offered when the task is posed. For purposes of our discussion here, we will act as if all of them have been given. We look at the solutions one by one to consider notation the teacher might use during the class discussion to record the child's reasoning and consider various aspects of how the overall discussion might proceed.

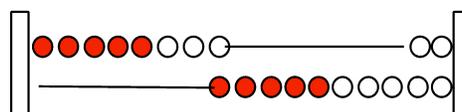
Louis' solution. I got 14. I broke the 6 into 2 and 4, put the 2 with the 8 to get 10, and added the 10 and the 4 to get 14.

Bus

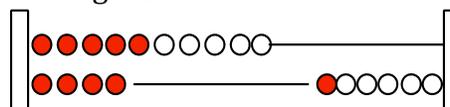


Louis' solution was to partition 6 into 2 and 4, combine the 2 with the 8 to get 10 and add on the remaining 4 from the 6 to get 14. Before we discuss teacher notation, notice that in his explanation, Louis reported *what* he did but he did not explain *why*. Why did he partition 6 into 2 and 4 and not 1 and 5 or 3 and 3? The teacher and many others in the class will know that he did so because he wanted to get a 10. And he wanted to get a 10 because he, and many others in the class, knows the result of adding a single digit number to 10. But there will be some children in the class for whom it is important to explicitly state this rationale.

The AR analog to this solution is to have 8 beads on the top rod



and add 6 by first filling up the top rod—that involves moving 2 beads to the left on the top rod—and then adding the remaining 4 on the bottom rod.



In discussing this AR solution in Part II of the sequence, we referred to this as the filling-up-the-tens strategy. Louis' grouping strategy is a filling-up-the-tens strategy but is no longer prompted by the visual support of the AR. He now has the filling-up-the-tens strategy as part of his mental computational toolbox. The teacher might even remark, "Some of you might have thought about the beads and how you would move them. Louis thought about it without thinking of the rack. He just thought it in his mind."

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The notation the teacher can use here is the same as she used for the analogous AR solution.

$$\begin{array}{c}
 8 + 6 = \\
 \begin{array}{c}
 \diagdown \quad \diagup \\
 2 \quad 4 \\
 \hline
 10
 \end{array} \\
 10 + 4 = 14
 \end{array}$$

And, as we have stressed before, the discussion should include talking about how the notation “shows how Louis figured it out.”

Danielle’s solution. I put 4 of the 8 with the 6 to get 10 and then I added on the other 4 from the 8 to the 10 to get 14.

Bus

$$\begin{array}{c}
 \text{Bus} \\
 \text{8} \\
 + 6
 \end{array}$$

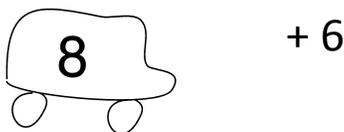
Danielle’s solution is much like Louis’ solution. However instead of partitioning 6, she partitioned 8. She explained that she wanted to make 10 but she thought of doing so by adding to the 6. Danielle also filled up the 10 but did it in a different way. Here again the discussion will include ensuring that everyone understands that getting a 10 was Danielle’s motivation and why getting a 10 is useful. Here we show notation the teacher can use to record Danielle’s thinking.

$$\begin{array}{c}
 8 + 6 = \\
 \begin{array}{c}
 \diagup \quad \diagdown \\
 4 \quad 4 \\
 \hline
 10
 \end{array} \\
 10 + 4 = 14
 \end{array}$$

Christian’s solution. I took 5 out of the 8 and 5 out of the 6. I added the two 5’s to get 10. I added the 3 left from the 8 with the 1 left from the 6 to get 4. 10 and 4 make 14.

Bus

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$$8 + 6$$

Christian's solution is another grouping solution based on partitioning numbers to get a 10. Here 10 is made up of two 5's so it is most likely that everyone in the class will easily make sense of this solution. Even so, it is important that the discussion emphasizes the reason for getting a 10. It is "easy" (for most students) to add a small number to 10. And it is important that the teacher use numeric notation to record Christian's thinking. Here we show one way to do so.

$$\begin{array}{r}
 8 \\
 \swarrow \searrow \\
 5 \quad 3
 \end{array}
 +
 \begin{array}{r}
 6 \\
 \swarrow \searrow \\
 5 \quad 1
 \end{array}
 =$$

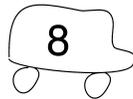
$$5 + 5 = 10$$

$$10 + 4 = 14$$

Another way is to draw a loop around the two 5's and show that they make 10, similar to notation we showed for the previous two solutions.

At this point, the teacher might choose to continue the discussion by asking the class to notice similarities and differences in the three solution methods. Each one is a grouping solution that is based on getting a 10 and then adding on the remaining amount. But each child used a different grouping to get the 10. Ensuring that the notation for all three solutions is visible simultaneously can facilitate this discussion. At this point the whiteboard (or screen) will have notation for three grouping solutions and will look like this.

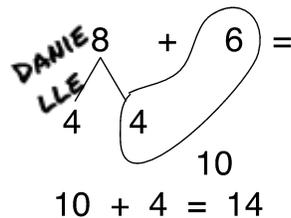
Bus


 $8 + 6$

$$\begin{array}{c} 8 \\ \swarrow \quad \searrow \\ 5 \quad 3 \end{array} + \begin{array}{c} 6 \\ \swarrow \quad \searrow \\ 5 \quad 1 \end{array} =$$

$$5 + 5 = 10$$

$$10 + 4 = 14$$


 $8 + 6 =$
 $4 \quad 4$
 10
 $10 + 4 = 14$

$$\begin{array}{c} 8 \\ \swarrow \quad \searrow \\ 5 \quad 3 \end{array} + \begin{array}{c} 6 \\ \swarrow \quad \searrow \\ 5 \quad 1 \end{array} =$$

$$5 + 5 = 10$$

$$10 + 4 = 14$$

The rationale for including this aspect of the discussion is to highlight the different ways efficient groupings can be achieved for this task and to further link numeric notation to the children's ways of reasoning. In this way the children's reasoning becomes increasingly about quantity whereas previously it was about beads on the rack.

Other solutions. The last three solutions given for this task are not grouping strategy solutions. Ben used a counting solution. In this class counting solutions are considered to be inefficient and are acknowledged but no discussion time is devoted to them. The teacher will thank Ben for his solution and might say, "Ben counted to figure it out. Who figured it out without counting?" The teacher will not write down anything to record his solution.

The solutions given by Joe and Sue are thinking strategy solutions. In each case the child used something he or she already knew to figure out the result. These involve higher-level reasoning and, as such, the teacher will devote discussion time to these solutions, ensuring that they are elaborated so others understand. For example, the teacher might ask, "Who knows what Joe is talking about? Manuel, what do you understand about how Joe figured this out?" We also encourage teachers to use numerical notation to record these thinking strategy solutions. They are easily notated by using number sentences as shown here.

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Joe: I know that 6 and 6 equal 12. So 8 and 6 are 2 more. I got 14.

Sue: I know that 8 and 8 are 16. So 8 and 6 are 14.

$$\begin{array}{l} \text{JO} \\ \text{E} \end{array} \quad \begin{array}{l} 6 + 6 = 12 \\ \text{So } 8 + 6 = 14 \end{array}$$

$$\begin{array}{l} \text{SU} \\ \text{E} \end{array} \quad \begin{array}{l} 8 + 8 = 16 \\ \text{So } 8 + 6 = 14 \end{array}$$

Tasks posed purely numerically. So far we have discussed tasks posed in a story context. Our purpose was to discuss solution methods students might use, to exemplify numerical notation the teacher might use to record children's grouping and thinking strategies, and to elaborate how discussing the notation can promote children's advancement to purely numerical reasoning. Tasks can also be posed purely numerically as shown here.

$$\begin{array}{l} 7 + 8 = \\ 16 - 7 = \\ 9 + _ = 16 \\ 14 - _ = 8 \end{array}$$

The intention is that these tasks are used in the whole class setting where children do not have paper and pencil but reason entirely mentally. The teacher poses one task at a time orally and also shows it on the whiteboard or screen for children to see as they think about how to figure it out. The solution methods we would expect for the first two tasks are the same as those we have already discussed in great detail for the analogous story context tasks. Consequently, here we will limit the discussion to the last two tasks, the missing addend task and the missing subtrahend task. Tasks of these two types can also be posed in story contexts but we have chosen to include our discussion of such tasks here.

Example 1. $9 + _ = 16$.

What do children do? The most primitive solution is to count on. However we would expect that almost no one would do so by this time in the instructional sequence. If a child does count on, the teacher will acknowledge this solution but not devote any discussion time to it. Instead she will focus on grouping and thinking strategy solutions. What might some of those be?

Amanda: 7. I thought of adding 1 to the 9 to get up to 10, and then adding on 6 more to get to 16.

Mark: I know that 10 + 6 make 16 so 9 + 7 is 16. My answer is 7.

Jose: I know that 8 + 8 equals 16 so 9 + 7 equals 16.

In the following paragraphs we discuss the type of notation might the teacher use to record each child's reasoning.

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Amanda used a grouping solution. The discussion of her solution will surely include talking about *why* she wanted to get to 10, how doing so made it “easy” for her to figure out what to do next. Here we show numerical notation the teacher might use to record Amanda’s reasoning. As the discussion progresses the teacher might draw a loop around the 1 and the 6 to highlight that she added 7 in all.

$$\begin{array}{r} 9 + 1 = 10 \\ 10 + 6 = 16 \end{array}$$

Mark used a thinking strategy solution. He reasoned that since $10 + 6$ equals 16, the answer here has to be 7, one more than 6. He didn’t provide all of those details but they will surely be elaborated in the discussion of his solution. The teacher can easily record his solution using number sentences.

$$\begin{array}{r} 10 + 6 = 16 \\ \text{so } 9 + 7 = 16 \end{array}$$

Likewise, Jose’s solution, which is also a thinking strategy solution, can be easily recorded using a pair of number sentences as shown here.

$$\begin{array}{r} 8 + 8 = 16 \\ \text{so } 9 + 7 = 16 \end{array}$$

We remark in passing that knowing the number combinations for 10, knowing the result of adding a small number to 10, and knowing doubles are the basic understandings that make these ways of reasoning possible.

How might children solve a missing subtrahend task at this point in the instructional sequence? Let’s use an example.

Example 2. $14 - \underline{\quad} = 8$

What do children do? The least sophisticated solution is to count down. Let’s take a look at non-counting solutions since they are the focus of this instructional sequence.

Marcus: I took off 4 to get down to 10. Then I took off 2 more to get down to 8. My answer is 6.

Angel: 6, because I know that $8 + 6$ equals 14. So 14 take away 6 equals 8.

Sam: I thought of how much I have to add to 8 to get back to 14. 8 and 2 more makes 10, and 4 more makes 14. So 14 take away 6 has to equal 8.

Marcus used a grouping down solution. He took off 4 to get down to 10 and then took off 2 more to get down to 8. His solution is easy to record using these two number sentences.

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The teacher can choose to draw a loop around the 4 and the 2 to emphasize that these comprise the 6 he took away.

MARCUS

$$\begin{array}{r} 14 - 4 = 10 \\ 10 - 2 = 8 \end{array}$$

Angel's solution is based on her understanding of the inverse nature of addition and subtraction. This pair of number sentences can be used to easily record her solution.

ANGEL

$$\begin{array}{l} 8 + 6 = 14 \\ \text{so } 14 - 6 = 8 \end{array}$$

Sam's solution involves both an awareness of the inverse nature of addition and subtraction and a grouping strategy. His grouping strategy was based on starting with 8 and filling up the ten by adding 2, then adding on 4 more to get to 14. Here again, number sentences are a very useful way to record his thinking.

SAM

$$\begin{array}{l} 8 + 2 = 10 \\ 10 + 4 = 14 \\ \text{so } 14 - 6 = 8 \end{array}$$

All three of these solutions represent sophisticated ways of reasoning that emerged from earlier intensive work with the arithmetic rack.

We want to make one additional remark about the notation shown here. In Part II of the structuring numbers sequence, when the AR was still an integral part of the tasks, we recommended that the teacher not notate missing addend and missing subtrahend solutions. But here we suggest notating these solutions. Why the difference? The difference lies precisely in the fact that the rack is not involved in the reasoning here. Therefore, here the teacher is only attempting to record the quantities involved. However, when the rack is used the teacher is attempting to indicate which beads were moved. Our extensive experience on this point is clear. Teachers and children have much difficulty figuring out how to use numerical notation to show the movement of beads for missing addend and missing subtrahend solutions. Inevitably everyone is left confused in those situations. But here, when only quantities are involved, there is no confusion.

Focus on Written Numerical Notation – “Read My Mind” Activity

Now we turn to an activity that focuses specifically on written numerical notation.

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We were introduced to this activity by a creative first grade teacher we worked with who used it very productively with her class. She called it “read my mind.” She would write a number sentence on the whiteboard and show some grouping strategy notation. Then she would say to the class, “Read my mind. How did I figure this out?”

Example 1. Here is an example of the “read my mind” activity. The teacher wrote the number sentence $6 + 7 = 13$ on the white board along with the notation shown here. Then she asked the class to try to explain how she figured it out.

$$\begin{array}{ccccccc}
 6 & + & 7 & = & 13 \\
 \swarrow & & \swarrow & & \\
 5 & 1 & 5 & 2 & \\
 \underbrace{\hspace{1.5cm}} & & & & \\
 10 & & & & \\
 10 + 3 = 13 & & & &
 \end{array}$$

As you can imagine, first graders loved this activity. And they quickly became adept at reading the teacher’s mind.

In this case, for example, one or more children explained that she split the 6 into 5 and 1 and the 7 into 5 and 2, put the 5’s together to make 10, then added the 1 and the 2 to get 3 and the 3 to the 10 to get 13.

Children used varied terminology. For example, some referred to “splitting the 6 and 7”. Some talked about “taking 5 out of 6 and 5 out of 7”. Some talked about “breaking the numbers, breaking 6 into 5 and 1 and 7 into 5 and 2.” Some even used the terminology of partitioning. “You partitioned 6 into 5 and 1 and 7 into 5 and 2.” This varied terminology reveals that the children were talking in ways that made sense to them. They each had their own ways of thinking about the activity of partitioning. More important, the language used here reveals that by now the children had developed conceptions of these numbers as objects in a mathematical reality. They can take a number, such as 6, and do something to it. They can break it up, split it. It is an object that they can manipulate. It is an object in their mathematical world.

You can see that this type of task can be easily used in isolation whenever there is as little as 5 minutes of time available. However, it can also be combined with other similar tasks to further emphasize the numerical notation. For example, suppose this task is followed by the one show in example 2.

Example 2.

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$$\begin{array}{c}
 \begin{array}{c}
 \text{6} + \text{7} = 13 \\
 \text{4} \quad \text{3}
 \end{array} \\
 10 \\
 10 + 3 = 13
 \end{array}$$

Here the number sentence is the same, $6 + 7 = 13$. But the numerical notation that is used to show how the teacher reasoned differs. Once again the teacher asks the class, “Read my mind. How did I figure it out this time?” The children’s task is to make sense of the notation. In this case, to understand that the teacher used a filling-up-the-tens strategy by combining 4 of the 7 with the 6 to get 10, and then adding on the remaining 3 to get 13.

The read my mind activity is intended to be used only after the grouping strategies are firmly developed by most children in the class. The emphasis is on interpreting the numerical notation. For this reason we recommend that this activity be used very close to the end of the instructional sequence and only a few times.

Timeline for Structuring Numbers Instructional Sequence

In this video we have described in detail Part III of the structuring numbers sequence, becoming efficient with additive tasks without the rack. Here we show a timeline for the entire sequence.

Timeline for Structuring Numbers Instructional Sequence

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	
PART I Becoming efficient with reading the rack										
Introductory activities	■									
Show this many people on the bus	■									
How many people – make configuration specified	■									
Rack flash			■							
Rack bingo			■	■	■					
Rack flash variation			■							
PART II Becoming efficient with adding and subtracting using grouping strategies prompted by the rack										
Reasoning by moving the beads					-----					
Anticipation activity						-----				
Reasoning without moving the beads								-----		

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<i>Imagination bingo</i>	----- ■ ■
PART III Becoming efficient with additive tasks without the rack	
Additive tasks without the rack	-----
Focus on written numerical notation	-----

As this timeline shows, Parts II and III of the sequence overlap. The overlap is deliberate. The intention is that activities from Part III are introduced and begin to be used while the later activities from Part II are still being used. For example, a lesson might consist of several tasks where the teacher uses her rack to pose the tasks that students solve entirely mentally followed several tasks posed in a story context entirely without the rack.

Furthermore, we show the timeline for a total of nine weeks. Yet many teachers continue to pose tasks in a story context or purely numerically for some time beyond the 9 weeks shown here. In this way students continue to firmly establish thinking and grouping strategies as part of their conceptual basis for solving additive problems for numbers to 20. At the same time, children increasingly come to “just know” the addition and subtraction combinations for numbers to 20.

Throughout the three video groups that describe the Structuring Numbers instructional sequence in detail we have attempted to explain how children’s understandings of number and number relationships develop. As these understandings develop children learn to think and reason mathematically. While they are becoming proficient with number facts for numbers to 20, they are learning much more. They are developing mathematical realities, mathematical worlds that they can navigate in meaningful ways. They are becoming mathematical thinkers.

There is a fourth video group, with an accompanying set of notes, that talks about the teacher’s role in successfully implementing the Structuring Numbers instructional sequence. This group is devoted to two main issues. One relates to planning lessons, including selecting which activities to combine to comprise a lesson and, for any specific activity, selecting which tasks to pose to enhance the learning opportunities for students. The second issue is how to orchestrate productive class discussions. Examples are used to illustrate these issues. This fourth video group, and the accompanying notes, can be viewed or read meaningfully at any time. It is not necessary to first view all of the video groups that describe the details of the instructional activities or first read all of the notes that accompany them.