Notes to Accompany Video Group 2: Structuring Numbers Part II

Part II: Becoming Efficient With Adding and Subtracting Using Grouping Strategies Prompted by the Rack

These notes accompany the second of a set of videos devoted to the Structuring Numbers Instructional Sequence. In these notes we discuss Part II of the sequence, becoming efficient with adding and subtracting using grouping strategies prompted by the rack. In many ways this is the heart of the sequence. More instructional time will be devoted to this part than to Parts I and III. In this part of the sequence children advance from first using their arithmetic racks as they solve addition and subtraction problems, posed within the double-decker bus scenario, to using a visual image of the rack as a means for reasoning mentally. By the end of Part II many students are able to reason using only mental imagery.

The ultimate purpose of this part of the sequence is that students develop the conceptual basis for solving addition and subtraction tasks, including missing addend and missing subtrahend tasks, that involve numbers to 20 by using grouping rather than counting strategies. By grouping strategies we mean ways of reasoning based on partitioning the numbers involved in the task into groups, based on doubles, 5's and 10's, and combining them in efficient ways.

We begin by listing the instructional activities that comprise Part II. The remainder of these notes will discuss each activity in detail, including how it is used, its instructional intent, how children might engage in the activity, and what they might learn from doing so. In addition, these notes include pragmatic information for teachers.

Instructional Activities

Here we list the instructional activities that comprise Part II of the instructional sequence. In these notes we discuss each one in detail.

1. Introductory activity
2. Reasoning by moving the beads
3. Anticipation activity
4. Reasoning without moving the beads
5. Imagination Bingo

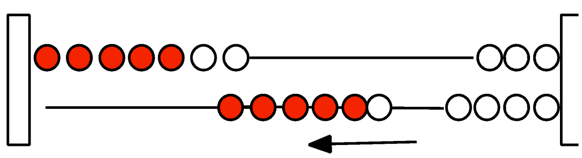
The tasks posed in Part II are additive in nature. In each case, the task involves beginning with some number of people on the bus and then changing that amount. Either more people get on the bus or some get off the bus. The task is to figure out how many people are on the bus after the change has taken place or, in some cases, if the number of people on the bus after the change is known, to figure out how many people got on or off the bus.

As in Part I of this sequence, Part II begins with an introductory activity that ensures that children understand how to use the rack to show what happens in the scenario—how to show that more people get on the bus or how to show that people get off the bus.

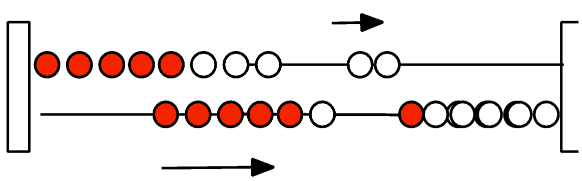
After that initial activity, there are two major segments to this part of the sequence. In the first of these segments, reasoning by moving the beads, children use their racks to complete the tasks. In the second major segment of Part II, reasoning without moving the beads, children shift from using their racks to complete the tasks to reasoning with the support of a visual image of the rack. The anticipation activity is a transition activity that supports making this shift. And imagination bingo is an activity involving little discussion that is appropriate after most children can reason easily without moving the beads.
The goal is that by the end of this part of the sequence, many children will have developed rack-related number relationships that they can use to solve additive tasks for numbers to 20. These rack-related number relationships form the basis for grouping strategies.
After discussing each of these activities we will turn to a number of pragmatic issues, including productive number choices, recommendations for how to show the initial configurations, and a detailed discussion of pedagogical notation that supports reasoning numerically.

1. Introductory activity. The tasks posed in this part of the sequence involve reasoning about how many people are on the bus after more have gotten on or some have gotten off. Initially children enact the change on their racks. Therefore it is important that they understand how to use the rack to do so. Our experience is that ways of showing the change in the number of people on the bus are not automatic for all children. Therefore, it is useful, actually necessary, to take the time to clearly establish conventions at the outset. A brief conversation with the class typically suffices. The sole purpose of this introductory activity is to establish these conventions.

Begin by explaining that to show more people get on the bus you move additional beads to the left. In this example, the motion shows that 6 more people got on the bus, all of them on the lower deck. (Initially there were seven people on the bus, all on the upper deck.)



Next explain that to show some people get off the bus you move beads from the left to the right. In this example, the motion shows that 8 people got off the bus, 6 off the lower deck and 2 off the upper deck. (Initially there were 16 people on the bus, 10 on the upper deck and 6 on the lower deck.)



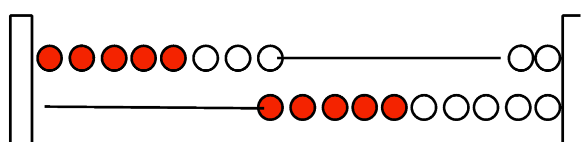
2. Reasoning by moving the beads. Activity 2, reasoning by moving the beads, is one of the main activities in Part II of the SN sequence. This activity will be used over and over, many times.

Each task is posed in two parts.

1. The teacher shows (not flashes) a configuration on her rack to represent the number of people on the bus.
   - The children show the same configuration on their racks.
   - The class agrees on how many people are on the bus.
2. Next the teacher states how many more people get on or how many get off the bus.
   - Children indicate the change on their racks in any way they choose, i.e., they decide for themselves which beads to move. Then they use what they have on their racks to figure out how many people are on the bus now.
   - There is a subsequent class discussion.

Let’s look at an example.

Example 1. The task the teacher has in mind is 8 + 6 but she does not pose the task by saying that. Instead, the teacher shows the initial quantity. Here we show the configuration we have chosen for this example.



Children show the same quantity on their racks in the same way. A brief class discussion establishes that there are 8 people on the bus. By this time in the sequence this discussion will be very brief. The sole purpose of the discussion is to ensure that everyone in the class agrees that there are 8 people on the bus.

Now the teacher poses the rest of the task, “6 more people get on the bus.” The children show that by moving 6 more beads from right to left, doing so in any way they choose. Then the children’s task is to use the resulting configuration on their racks to figure out how many people are on the bus now.

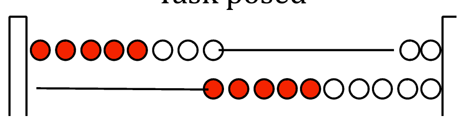
The subsequent class discussion focuses both on the final result and how the child figured it out, and on which beads the child moved to get to that result. Notice that once the child has moved the beads on his rack to show the change, the task he has now is to “read” his rack, and, ideally, to do so efficiently. That’s exactly what Part I of this sequence was about—learning to read the rack efficiently. So, by now many students will be able to do so. However, it is important that the discussion also includes talking about which beads the child moved and why, since it is that aspect of the discussion that has the potential to relate to grouping strategies.

Before we discuss what children do, I want to remark that it is important for the teacher to allow sufficient time for students to figure out the task before beginning the discussion. In our experience teachers new to this instructional sequence frequently begin the discussion before children have had sufficient time to do so. When the discussion begins too quickly, only the most sophisticated children will have actually completed the task. The result is that the other children then might not finish thinking through the task. And, of course, that means that the activity is less productive for their learning.

How do you ensure that children have had enough time? One way is to ask them. “Raise your hand if you want a little more time to figure this out.” Or to privately say to individual children, “Have you figured it out yet? Would you like more time?” Just to emphasize this point a little more, research shows that teachers often call for a response within 5 to 10 seconds after posing a question. But if we pose a task that requires some figuring out, then surely we need to give children time to do so.

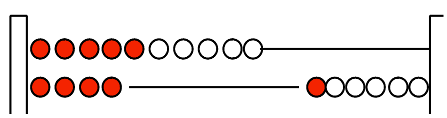
What do children do? Returning to our example, everyone starts off with 8 beads moved to the left on the top rod. To show that 6 more people get on the bus children have to move 6 more beads to the left. There are no restrictions about how they do so. They can move beads on the top rod, on the bottom rod, or some on each.

Task posed



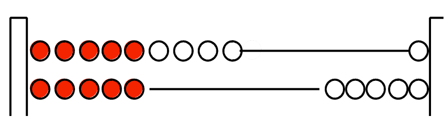
6 more get on the bus.

For example, here they could move 2 beads on the top and 4 on the bottom, as Andi did,



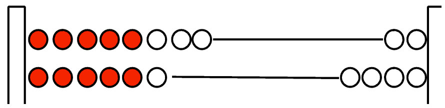
Andi

1 on the top and 5 on the bottom, as Brian did,



Brian

or all 6 on the bottom, as Carlos did.



Carlos

Once children have moved the beads on their racks and have been given sufficient time to figure out what they have now, the teacher engages the class in a discussion.

Before we talk about how the discussion might proceed let's remind ourselves of the instructional intent of this part, Part II, of the SN sequence. It is:

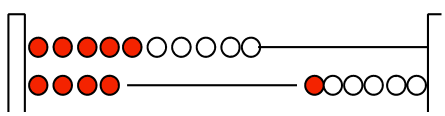
- To foster students' use of grouping strategies for additive tasks as an outgrowth of the way they move the beads to show the change in quantity.

Or to say it another way,

- The goal is that children develop ways to solve additive tasks involving numbers to 20 using grouping strategies that have been prompted by the color configurations of the beads.

With this in mind, the teacher's agenda is to foster children's conceptual development to that end.

Andi. Let's look at the way Andi moved the beads and consider how that might contribute to developing grouping strategies. After moving beads to show that 6 more people got on the bus, Andi's rack looks like this.



Andi

The teacher calls on her to explain. Remember that Andi is expected to explain not only how many are on the bus now and how she figured it out, but also which beads she moved and why.

Because of the emphasis in Part I of this sequence on reading the rack efficiently, Andi might easily figure out the total is 14 because she sees 10 beads on the top, 5 red and 5 white, and 4 red on the bottom. 10 and 4 make 14. The crux of the discussion comes in as Andi explains which beads she moved and why. For example, *why* did she move the 2 beads on the top? She might not have a reason for why. But she might say that she did so to fill up the top. Then the question is why did she want to fill up the top? Well, if you fill up the top you have a 10. Having a 10 is advantageous if you "just know" that 10 and 4 make 14. And this relationship between 10 and the related teen numbers is something that many children will have developed in Part I of this sequence. As Andi elaborates why she moved 2 beads to fill up the top, either she or some other student in the class will call attention to the usefulness of 10. The teacher can further highlight how "filling up the 10" made the resulting configuration easy to figure out. In this way, Andi's way of moving the beads becomes the basis for beginning to reason by grouping to fill up the 10s. Some children that moved the beads in the same way but did not think in advance of doing so to create a 10 might now become explicitly aware of how doing so made it "easy" to read off the result. Still other children that have not yet have become proficient at reading the rack efficiently have the opportunity to continue that development, including developing the understanding that 10 and 4 more makes 14.

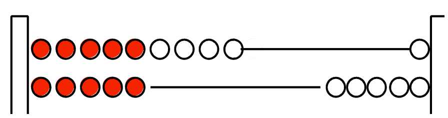
Of course for the discussion to be comprehensible, it is important that the teacher shows the solution on her own rack so children can see it as the discussion takes place. Further, I want to emphasize that for these discussions to be effective, they must be thorough. The discussion needs to include talking about all of the things we have mentioned here:

- How Andi added the 6 as 2 and 4.
- Where on the final configuration we can "see" the 2 and the 4.
- How the 10 (beads on the top rod) are made up of the original 8 and 2 of the 6 added.
- How moving the beads in that way made it "easy" to figure out 14.
- How just knowing that 10 and 4 more make 14 makes this way efficient.

Discussions of this type take time. The class might spend 5 to 10 minutes discussing Andi's solution. The time is well spent.

Over the course of this part of the instructional sequence, the teacher will pose many tasks of this type and have many discussions that link moving the beads the way Andi did to filling up the tens. Over time, more and more students will move the beads purposefully to that end.

<u>Brian.</u> Brian showed 6 more getting on the bus by moving 1 bead on the top rod and 5 beads on the bottom rod to the left. Here we see the final configuration on Brian's rack.



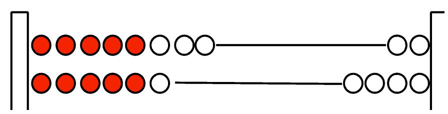
Brian

His rack is also "easy" to read because, like Andi, the way he moved the beads resulted in a 10. He might "just know" that 10 and 4 are 14 or he might count on by ones from 10 to get 14. This part of the discussion will be relatively straightforward in the class for everyone by now. However, there is much more to say about how he moved the beads to get 10. Where does the 10 come from? It comes from the two sets of 5 red beads. The discussion will include talking about how one of the 5's comes from the 8 and the other 5 comes from the 6. Where does the 4 come from? The four is from the white beads. Three of those are from the 8 and the other white bead is from the 6 that were added.

Notice how this part of the discussion includes talking about beads some of the time and about quantity at other times. This subtle aspect of the discussion is precisely how it functions to facilitate children's linking the way the beads are moved to grouping quantities. The 10 came from combining 5 from the 8 with 5 from the 6. This way of talking about Brian's solution will *eventually* result in children being able to solve the task 8 + 6, presented purely numerically, by partitioning 8 into 5 and 3, partitioning 6 into 5 and 1, combining the 5's to get 10, the 3 and 1 to get 4, and the 10 and 4 to get 14. We say *eventually* because that way of reasoning is the goal of Part II of the SN sequence. It is the end point. Here we see the genesis of that way of reasoning. It is based on moving the beads. And it is the way the beads are structured on the rack that makes that possible.

We remind teachers that developing these ways of reasoning requires conceptual development. We are not teaching certain ways of reasoning. We are creating the possibility for children to develop ways of reasoning that grow out of their actions and attempts to make sense of those actions. Not all children develop the same ways of reasoning. And, since conceptual development is based on what the child already knows and understands, children differ substantially in the amount of time required. As we said when discussing Andi's solution, many tasks of this type will be posed and discussed before Part II of the sequence concludes.

Carlos. There's yet one more way children could have moved the beads to solve the task posed. That way is shown by Carlos' solution method. Carlos moved 6 beads to the left on the bottom rod to show 6 people getting on the bus. Carlos' rack looks like this.



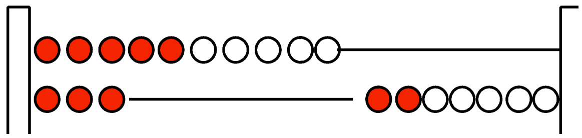
Carlos

Once again we have a rack that can be read efficiently as 10 and 4, for a total of 14. The difference between Carlos' solution and Brian's solution is in the 4 white beads. Brian had 4 white beads, all on the top rod. Carlos has 3 on top and 1 on the bottom. In a sense Carlos' solution will be easier to talk about in the class discussion because the 6 beads that he added are shown all on the bottom rod. For this reason it will most likely be easier for children to "see" how the 6 that were added can be thought of as 5 and 1 and the original 8 as 5 and 3. Otherwise the discussion will be similar to that for Brian's solution. The 10 came from combining the 5 red beads on the top rod (the 5 from the 8), with the 5 red beads on the bottom rod (the 5 from the 6). The 4 came from the 3 white beads on the top (from the 8) with the white bead on the bottom (from the 6). It is the color configuration of the beads that contributes to thinking of 8 as 5 and 3 and 6 as 5 and 1. And it is precisely because Carlos added all 6 beads on the bottom rod that this way of "seeing" the 8 and the 6 is possible. Carlos' solution contributes to developing the basis for reasoning about adding 8 and 6 by partitioning each number into 5 and what remains, adding the two 5's and then adding on the rest.

Tasks like this where children use their racks and actually move the beads are critical to forming the conceptual basis necessary for later reasoning purely numerically by decomposing and recomposing quantities using grouping strategies. In the example we have presented, each possible way of moving the beads led to creating a 10 that could then be linked to an efficient grouping strategy. There are examples where some ways of moving the beads do not lead to results that can be linked to efficient grouping strategies. We will come back to this point later and give an example.

Example 2. Now let's consider a subtraction example. The task the teacher has in mind is 13 – 6 but she does not pose it that way.

She shows 13 on her rack as seen here.

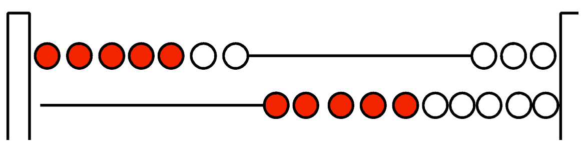


Children show the same quantity on their racks in the same way. A brief class discussion establishes that there are 13 people on the bus.

Then the teacher says that 6 people get off the bus. Each child shows the change on his rack in any way he chooses and then uses the resulting configuration on his rack to figure out how many people are on the bus now. The subsequent class discussion focuses on which beads the children moved to show that 6 people got off the bus and how they figured out how many people are on the bus now.

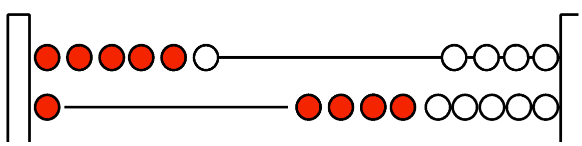
What do children do? They move 6 beads from left to right in any way they choose. In this example there are four possibilities. They are:

- Move the 3 beads on the bottom to the right, that is, empty the bottom, and move 3 beads on the top to the right, as Darius did.



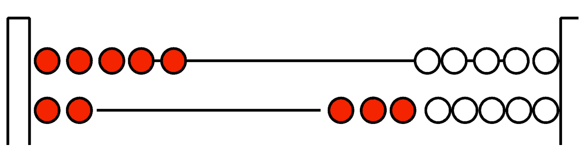
Darius

- Move 2 beads on the bottom to the right, and move 4 on the top to the right, as Xavier did.



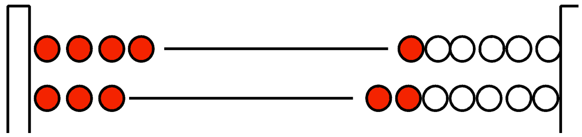
Xavier

- Move 1 bead on the bottom to the right, and move 5 beads on the top to the right, as Anna did.



Anna

- Move 6 on the top to the right, as Gabriella did.

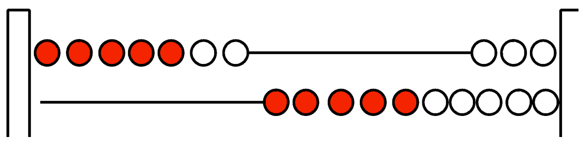


Gabriella

The resulting configuration in each case is shown above.

We will discuss each of these ways in turn and consider the potential of each solution to promote children's development of grouping strategies for subtraction.

Darius. Let's look at the way Darius moved the beads and consider if that might contribute to developing grouping strategies for subtraction, and if so, how. Darius moved the 3 beads on the bottom to the right, that is, he emptied the bottom, and he moved 3 beads on the top from left to right. Darius' rack looks like this.



Darius

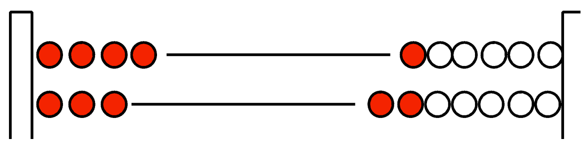
When called on to explain how he moved beads, how many people are on the bus now, and how he figured it out, what might Darius say? The resulting configuration of 5 red beads and 2 white beads on the top will be relatively easy by now for everyone in the class to figure out as 7. Darius might say, "There are 7 because 5 and 2 more make 7." To describe how he moved the beads he might say that he moved all of the beads that were on the bottom to the right (emptied the bottom) and also moved 3 beads on the top to the right. Notice that there is nothing explicit in what he has said so far that points to a grouping strategy. That is why it is important that the discussion continue to include Darius' reasons for moving the beads as he did. *Why* did he empty the bottom?

As Darius elaborates why he emptied the bottom, he or others might remark that in doing so he got down to 10. Then he took the rest of the beads away from the ten (on top). Why might he want to get down to 10? Maybe he just wanted to empty one of the rods. However, it is also the case that by now 10 is a very friendly number for most students so he might have wanted to get down to 10.

From a mathematical point of view, Darius' solution can be thought of as revolving around the relationship between a teen number and 10, in this case that 13 is 10 and 3, and also on children's familiarity with the combinations that make 10. (As an aside, those children that yet do not know the combinations that make 10 have further opportunities to develop those understandings through discussions of tasks like this one.) In other words, Darius' way of moving the beads can be the basis for an efficient grouping strategy, first grouping down to 10. Notice how this grouping strategy is essentially the reverse of the filling up the 10s strategy that is implicit in the solution Andi used on the addition example we discussed earlier.

One function the teacher serves is to ensure that the discussion includes talking about these number relationships and how they relate to the way Darius moved the beads. That is an essential part of the discussion. As children talk about both beads and quantity represented by the beads, they begin to develop the basis for eventually being able to reason by using strategies based on number relationships involving 10.

Gabriella. Let's consider another solution. Gabriella's solution was to move all 6 beads from left to right on the top. Her rack looks like the one shown here.



Gabriella

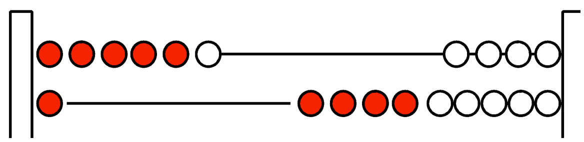
When asked how she moved the beads, how many are on the bus now, and how she figured it out Gabriella might say, "I took all 6 away from the top. Now there are 4 on the top and 3 on the bottom. That makes 7."

As with Darius' solution, there is nothing explicit in Gabriella's remarks so far that points to a grouping strategy. But her remarks suggest how the discussion might proceed to bring a grouping strategy to the forefront. We know that one of the typical ways sophisticated students reason to solve a task such as 13 – 6 is to think of the 13 as 10 and 3, take all 6 away from the 10, add the 4 that remains to the 3 from the 13 to get 7. That is an efficient grouping strategy that is based on a comprehensive understanding of 10—the way 10 relates to the teen numbers and the combinations that make 10. Gabriella's way of moving the beads directly mimics that grouping strategy, though at this point in time Gabriella's reasoning is almost surely driven by how she moved the beads.

It becomes the teacher's responsibility to ensure that the discussion includes talking not only about the beads but also about the quantities they represent and the number relationships involved. For example, here a legitimate question is, "Where are the 6 that Gabriella took away?" Gabriella, or others will say that she took 6 from the top. Others will say she took 6 away from the 10. She didn't take any away from the 3. Notice this shift from talking about the beads to talking about quantity. Someone will likely say that Gabriella put the 4 beads on the top with the 3 beads on the bottom to get 7. Someone else might say, "Gabriella took 6 away from the 10 and that left her with 4. She put the 4 together with the 3 on the bottom to get 7." These seemingly slight variations in what children say are important because they indicate how children shift between talking about the beads (how the beads were moved and what they see on the rack) and talking about relationships between quantities. And this dual way of talking is precisely how moving the beads on the rack to solve tasks over time gradually evolves into reasoning by using grouping strategies.

Over the course of this part of the SN sequence, using this activity and those that we will describe next, the majority of the students will develop 5-, 10-, and doubles-referenced number relationships for numbers to 20. These relationships will result in their being able to reason flexibly using grouping strategies. But this development takes time. It represents an evolution in the children's thinking and reasoning.

Xavier. Xavier's solution was to move 2 beads on the bottom and 4 beads on the top from left to right. His rack is shown here.



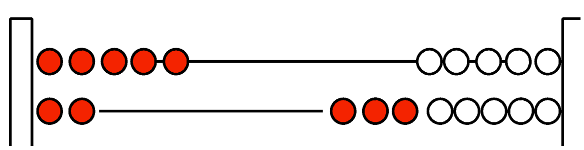
Xavier

As Xavier explains how he solved the task he will talk about how he took 2 beads off the bottom and 4 beads off the top. Now he can see 6 on the top and one more on the bottom. That makes 7.

As with the two previous solutions, it is relatively easy to figure out the 7. As the discussion continues, children will talk about where the 6 are that Xavier took away. He took 2 of the 6 away from the 3 and he took the other 4 of the 6 away from the 10. These remarks link Xavier movement of the beads with quantity. So here again the discussion involves shifts from talking about beads to talking about quantity. However, there is a big difference between Xavier's solution and the two we have discussed so far.

From a mathematical perspective, Xavier's solution involved thinking of 13 as 10 and 3, taking away 2 from the 3 and taking the remaining 4 from the 10. He then combined the 1 and the 6 that remained to get 7. This is also a grouping strategy in the sense that Xavier partitioned the numbers involved, subtracted the amount needed, and then recombined the quantities that were left. However, it is not an efficient grouping strategy.

Anna. Likewise, Anna's solution method, which was to take 1 bead off the bottom and 5 beads off the top, seen from a mathematical perspective, is not efficient.



Anna

Described mathematically, Anna thought of 13 as 10 and 3, took 1 away from the 3 and took the remaining 5 that needed to be taken away from the 10. She then combined the 1 and the 5 that were left to get 6. The discussion of Anna's solution will no doubt also include talking about beads and about the quantities they represent. But the grouping she used is not efficient.

We will come back to these two solution methods later to explain further why we say they are inefficient. While in both cases the discussion will include talking both about beads and about quantity, these ways of moving the beads will eventually drop out because they are not efficient. Here's what I mean by saying that they will eventually drop out. As tasks of this type are used over and over in many lessons, children begin to anticipate what the final configuration of the rack will be even before they move the beads. And as they do so, children increasingly begin to use the most efficient ways. And that leads us to the next instructional activity, one we call the anticipation activity.

3. Anticipation activity. The anticipation activity is conducted as follows:

- The teacher shows the initial quantity on her AR.
- Children show the same on their ARs.
- The teacher says how many people get on or off the bus.
- Children do *not* show the change on their racks but think about how they would do so and what the result would be.

This activity is similar to the "reasoning by moving the beads" activity. But there is one big difference. The difference is that this time after the teacher tells how many people get on or off the bus, the children are asked to *think about* how they would move the beads to complete the task but to *not actually move them*.

Specifically, the teacher shows the initial number of people on the bus on her AR. Children show that same configuration on their racks. Then the teacher says how many people get on or off the bus. But this time, instead of showing that change on their racks children are asked to **think about** how they would move the beads and how many people would be on the bus then. In other words, after beginning the task using their racks, children are asked to only imagine completing the task. They have to anticipate what the result would be.

We ended our explanation of the previous activity, reasoning by moving the beads, by saying that as tasks of that type are used repeatedly, children begin to anticipate what the final configuration of the rack will be even before they move the beads. In this anticipation activity the children are specifically asked to not complete the task with the beads but to do so only mentally. Crucially, they do move the beads on their racks to show the initial amount. In our experience children often touch or finger the beads they would move were they to complete the task using the rack. A few children are not able to complete the task only mentally but do so by moving the beads anyway. These children are participating at a level that is appropriate for them.

Class discussion of the children's solutions is an important part of this activity. It is through the discussions that children link the way they move the beads to quantities the beads represent. And it is this linkage that forms the basis for reasoning using grouping strategies. These discussions will be similar to those we described for the previous activity so we do not repeat them here.

Teachers that use this sequence successfully typically make gradual shifts in the activities they use. For example, after several days of using the reasoning by moving the beads activity, teachers pose several tasks of that type and then shift within the same lesson to the anticipation activity. Doing so provides opportunities for the more sophisticated children to continue to advance while the less sophisticated students continue to complete the tasks by moving the beads. The shift is easy to make. After posing several tasks where children complete the task moving the beads, the teacher can simply say, as she poses the next task, "This time, instead of moving the beads just think about how you would move them and figure out what the result will be."

4. Reasoning without moving the beads. The reasoning without moving the beads activity is conducted as follows:

- Children do *not* have their racks to use.
- The teacher shows the initial configuration on her rack.
- Then she says how many people get on or off the bus.
- The students reason by looking at the teacher's rack and thinking about how to show the change and how many are on the bus now.

This fourth activity, reasoning without moving the beads, is similar to the anticipation activity except that this time the children do not have their racks to use. The teacher shows the initial configuration on her rack, which remains visible throughout the task. Then she says how many people get on or off the bus. The children have to reason entirely mentally but they have the visual support of the teacher's rack. After the class has had sufficient time to figure out the task, there is a class discussion. Children talk about which beads they imagined moving and how they used that information to figure out how many are on the bus now.

One or two children may need to have racks to use in order to participate meaningfully. However, if more than just a few children need to have racks to use, this activity is premature.

This activity will be used often over many lessons. And, as we discussed with the anticipation task, teachers typically combine this activity with previous activities within the same lesson. For example, the teacher may pose several anticipation tasks, then collect the children's racks, and then, or later in the lesson, pose several reasoning without moving the beads tasks.

It should be apparent by now that the activities listed in this part of the sequence form an evolution of sorts. As children participate repeatedly in tasks where they move the beads, they become able to anticipate the results without moving the beads themselves. Nevertheless, the visual image of the beads is present to support their reasoning.

Task types. There are four possible task types. They are:

- Addition tasks
- Subtraction tasks
- Missing addend tasks
- Missing subtrahend tasks

To this point we have discussed only addition tasks and subtraction tasks.

*Addition example***.** There are 8 people on the bus. 6 more get on.
Subtraction example. There are 13 people on the bus 6 get off.

In addition to these two task types, there are two other task types that can be used productively in the reasoning without moving the beads activity. These are missing addend and missing subtrahend tasks. Here we give an example of each task type.

Missing addend example. 7 people are on the bus. How many more need to get on to have 13 people on the bus?

For this task type the teacher uses her AR to show 7 people on the bus. The children's task is to use the image the teacher shows to figure out how many more people get on the bus to have 13. During the subsequent discussion children typically talk about which beads they would have moved, had they had their ARs, to show more people getting on the bus.

Missing subtrahend example. 15 people are on the bus. How many need to get off to have 9 people on the bus.

The teacher shows 15 people on the bus on her AR. Children use the image to figure out how many people have to get off to have 9 on the bus. Here again, children typically talk about which beads they would have moved, had they had their ARs, to show people getting off the bus. If they do not do so the teacher can ask them which beads they thought about moving.

5. Imagination bingo. In this activity, imagination bingo, the teacher shows an initial quantity on her rack and asks children to imagine some number of people getting on or off the bus. The children then cover the resulting number on their bingo cards. Children do not have racks to use for this activity.

There is little discussion with this activity. Typically teachers call on only one child to explain how he figured out the task. Keeping the discussion to a minimum makes it possible for the game to move at a reasonable pace. This activity is appropriate only after most of the children are able to reason without moving the beads.

Structuring Numbers Notes for Video Group 2
E. Yackel
Pragmatic Considerations

Now we turn to various pragmatic considerations involved in planning a lesson that uses the activities in this part of the SN sequence. These include:

- Deciding which tasks to pose.
- Deciding which numbers to use, i.e., how many are on the bus initially and how many people get on or off the bus.
- Deciding on the initial configuration to use in posing the task.
- Sequencing tasks, i.e. deciding which tasks to pose after one another.
- Planning how to recording children's solutions, i.e., anticipating the solutions children might use and thinking through in advance how to record children's solutions.

Which tasks to pose.

Productive number choices. The structuring numbers sequence is about numbers to 20, with a focus on the teen numbers, while the P&P sequence focused on numbers to 10. With that in mind, the tasks posed in these activities should involve going over 10.

Examples include:

- 7 people on the bus. 9 more get on.
- 4 people on the bus. 8 more get on.
- 6 people are on the bus. 6 more get on.

Each of these examples involves going over 10.
Examples *not* to use

- 3 people are on the bus. 5 more get on.
- 14 people are on the bus. 2 more get on.

These do not involve going over 10.

Subtraction examples include:

- 14 people are on the bus. 5 get off.
- 16 people are on the bus. 9 get off.
- 11 people are on the bus. 4 get off.

Again each of these examples involve going through 10.
But *not*

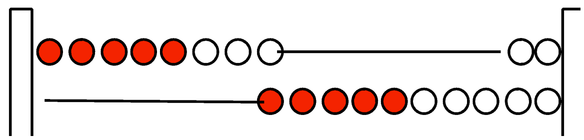
- 9 people are on the bus. 3 get off.
- 14 people are on the bus. 2 get off.

These examples do not involve going through 10.

Initial configurations. The second aspect of deciding which tasks to pose is to select the initial configuration to use in posing the task. This selection is made by anticipating possible moves children might make with their beads that promote grouping rather than counting solutions. Teachers we have worked with have told us that choosing the initial configuration from the large number of possible choices can be confusing for them. For example, if the teacher wants to use the task: *8 people on the bus, 6 get on*, there are many ways to show the initial 8. How might the teacher decide which configuration for the initial 8 is a good choice? In our work with teachers we have found that making specific recommendations that have proved useful relieves the teacher of this difficult decision.

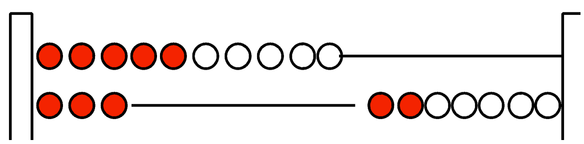
Here are our recommendations. For addition tasks, show the initial number (which will be a number under 10) entirely on the top rod.

Example: For the task 8 on the bus, 6 get on, show 8 as



Using this choice, children will typically show the second quantity either by filling up the ten on top and putting the rest on the bottom or they will put all of the second quantity on the bottom. Notice that these are the solutions given by Andi and by Carlos in the first example we discussed earlier.

For subtraction tasks, our recommendation is to show the initial number (which will be a teen number) as 10 on the top rod and the remainder of the number on the bottom rod.
Example:
For the task 13 on the bus, 6 get off, show 13 as



Using this choice, children will typically either empty the bottom rod (get down to 10) and take the remainder of the number off the top or they will take the entire amount off the top. These are the solutions given by Darius and Gabriella in the second example we discussed.

Certainly other choices for the initial configuration are possible. However by using these ways consistently, the teacher creates a situation for herself that is more manageable and one that guarantees that efficient grouping strategies will emerge in the discussions.

Sequencing tasks. Another aspect of lesson planning is deciding on the set of tasks to use within the lesson. The choices can have considerable impact on the learning opportunities the lesson provides. We recommend sequencing tasks to promote number relationships. Here we show two examples to clarify what we mean.

Example 1:
- First task - 5 on the bus, 7 get on.
- Second task - 12 on the bus, 7 get off.

Example 2:
- First task - 14 on the bus, 7 get off.
- Second task - 14 on the bus, 8 get off.

In example 1, the initial task is that 5 people are on the bus and 7 more get on. After a thorough discussion the next task the teacher poses might be 12 are on the bus then 7 get off. Invariably, someone in the class will notice how these tasks are alike. Children often say things such as, “Hey, that’s the same thing we just did.” That remark provides an opening for the teacher to say, “What do you mean? How are these the same?” And in the ensuing discussion the reversibility of addition and subtraction will be the focus, though neither the teacher nor the children will use that language. At the same time there will be children that solve the second task without thinking about relating it to the first. These children will talk about which beads they moved (or imagined moving) and the relationship between the beads moved and the quantities involved. This discussion is also productive.

The second example we show here is to pose 14 on the bus then 7 get off as a first task, followed by 14 on the bus then 8 get off. In this example some children might use their results from the first task to solve the second task and in this way the relationship between the quantities involved in the two tasks becomes a topic of discussion. Those children that did not notice the relationship on their own have an opportunity to begin to develop such ways of reasoning.

Teachers will surely be able to think of other examples of pairs of tasks that can promote number relationships. We don’t mean to imply that the teacher should make choices of this type every time. Rather, we are pointing out that such choices provide additional avenues for fostering a variety of number relationships.

Recording children’s solutions. An important aspect of Part II of the SN sequence is the use of numerical notation, both conventional and nonconventional, to record children’s solutions. This aspect should also be a part of the teacher’s lesson planning. Here we discuss three aspects of recording children’s solutions. These are:

- WHY record children’s solutions
- HOW to record children’s solutions - Recommendations for productive notation
- WHEN to begin recording children’s solutions

First we discuss why it is important to record children’s solutions. Next we make specific suggestions for notation that has proven to be productive for children’s learning by returning to the examples discussed earlier. Finally, we discuss when in the sequence it is appropriate to begin using notation. In earlier videos we have remarked that standard math curricula often introduce mathematical notation too early, well before children have developed early number concepts. The use of notation in Part II of the SN sequence comes in as a way to use mathematical symbols to express number relationships children are developing as they reason about additive tasks.

WHY record children’s solutions? Our reasons for recording children’s solutions are to:

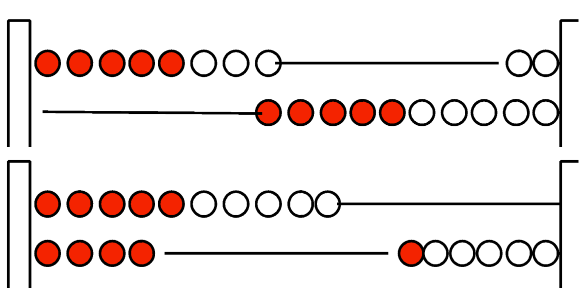
- Further link reasoning with the beads to grouping strategies.
- Link grouping strategies to number sentences and numerical notation.
- Facilitate communication.

In our discussion of the various activities in this part of the SN sequence we have emphasized the role of class discussion (except in the Imagination Bingo activity). The discussions are an important means of linking children's real or imagined actions with the beads with grouping strategies. This link typically arises as children talk about beads some of the time and about quantity some of the time. Further, as children become increasingly adept at anticipating solution methods, the numerical relationships between the quantities involved become more prominent. Another way to say it is that reasoning about the beads begins to drop out and reasoning about quantities comes to the fore. And, as we have said before, it is this shift to reasoning about quantities that makes it possible for grouping strategies to emerge as part of the conversation.

And this is where the use of numerical notation and number sentences can play an important role. The teacher can use numerical notation to "try to write down how ___ is thinking about it." In doing so, the teacher links symbolic notation to the child's reasoning and in this sense raises the level of abstraction. Now the grouping strategies that were prompted by the ways the beads were used are linked to numerical symbols. Further, as the teacher creates written records of several solution methods for the same problem, the class has an opportunity to compare and contrast the solutions, to take note of how they are alike and how they are different. Of course, children can compare and contrast solution methods from listening to the verbal descriptions alone. But verbal comments have no permanence. Written records allow more in depth discussions. They facilitate communication.

HOW to record children's solutions. In this section we present specific recommendations for notation that is productive or, in some cases, explain why we recommend not using numerical notation to record certain solution. To do so we look at specific examples of productive ways to record children's reasoning by returning to the tasks we discussed earlier. Earlier we discussed how various solutions gave rise to grouping strategies. Now we will consider how those grouping strategy solutions might be recorded numerically.

Notation for filling up the 10 grouping strategy. The first example we discussed earlier was an addition task: 8 people on the bus and 6 more get on. The first solution for the task was Andi's solution. She moved 2 beads on the top to the left and 4 beads on the bottom to the left. The initial configuration as the task was posed and the final configuration on Andi's rack are shown here as a reminder.



Initial configuration Andi's rack

The discussion focused on how Andi filled up the 10 (on the top rod) and then added the other 4 on the bottom rod to get 14. Andi just knew that 10 and 4 make 14. The grouping strategy implicit in Andi's solution is to fill up the 10. To initiate the use of notation, the teacher might say, "I'm going to try to write down something that shows how Andi thought about it." Then the teacher can write 8 + 6 and say, "Andi thought of adding the 2 (of the 6) to the 8 to fill up the 10 and then adding on the remaining 4 (from the 6) to the 10 to get 14. Ten plus four equals 14." Here we show two typical ways teachers use numeric notation to show Andi's solution.

8 + 6 =	8 + 6 =
2 4	2 4
8 + 2 = 10	10
10 + 4 = 14	10 + 4 = 14

The notation on the left shows the 6 partitioned into 2 and 4. The two number sentences show that Andi added the 2 (from the 6) to the 8 to get 10 and then added the remaining 4 (from the 6) to the 10 to get 14. The notation on the right also shows the 6 partitioned into 2 and 4 and uses a loop around the 8 and the 2 to show that those add to 10. The number sentence shows adding the remaining 4 (from the 6) to the 10 to get 14.

This type of notation is called "pedagogical notation." It isn't conventional notation but bears some similarity to conventional notation. We call it pedagogical notation because its primary purpose is as an aid in teaching, in pedagogy. It is a way of using numerals and number sentences to record the grouping strategy reasoning that is implicit in the way Andi moved the beads.

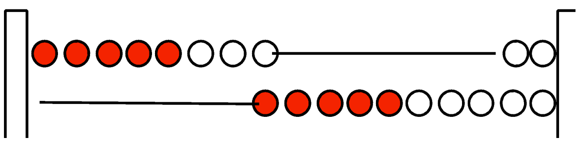
The same notation works to record the thinking of a child that moved the beads in the same way but who counted on by ones from 10 to get to 14. The teacher might say, "____ thought of it the same way but he counted on from 10 to figure out that 10 plus 4 makes 14. Andi just knew that 10 and 4 make 14." Since our goal is to advance grouping strategies, the teacher will not write anything down to record counting by ones.

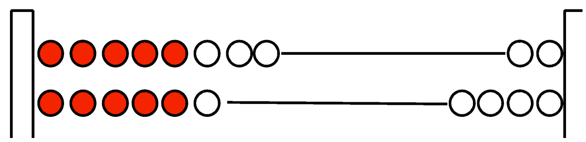
Further explanation and elaboration to ensure that the class makes sense of the notation and understands how it "shows how Andi figured it out," is important. In addition to facilitating communication, the notation serves to link reasoning that resulted from moving the beads to numerals and number sentences that capture the grouping strategy involved.

Our recommendation is that tasks of this type are posed and discussed for several days before the teacher begins to bring in notation. The numerical notation comes in as a way of recording the grouping strategies that reflect the *numerical relationships* involved.

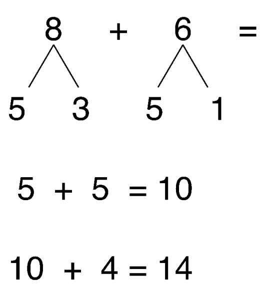
We also note that as the teacher uses number sentences, students become familiar with conventional mathematical language. For example, the number sentence 10 + 4 = 14 can be read as "10 and 4 make 14 or as 10 plus 4 equals 14." By referring to number sentences such as this in these two ways, as the teacher did in this example, she naturally introduces conventional mathematical language and links it to the children's informal ways of expressing their reasoning.

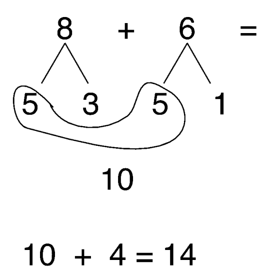
Notation for combining 5's grouping strategy. The second solution for the task that showed 8 people on the upper deck of the bus with 6 more getting on was Carlos' solution. He moved 6 beads on the bottom to the left. The initial configuration when the task was posed and the final configuration on Carlos' rack are shown here as a reminder.


Initial configuration


Carlos' rack

The discussion focused on how Carlos figured out 10 by combining 5 from the 8 beads on the top and 5 from the 6 beads on the bottom. Carlos then just knew that 10 and the remaining 4 beads (the white ones) make 14. Again, to initiate the use of notation, the teacher might say, "I'm going to try to write down something that shows how Carlos figured it out." Then the teacher can write 8 + 6 and say, "Carlos thought of adding the 5 from the 8 to the 5 from the 6 to get 10. That's all the red beads. Then there's 3 left from the 8 and 1 left from the 6 to add on. That's 4 more. 10 and 4 equals 14." Here we show two typical forms of numeric notation teachers use to show Carlos' solution.





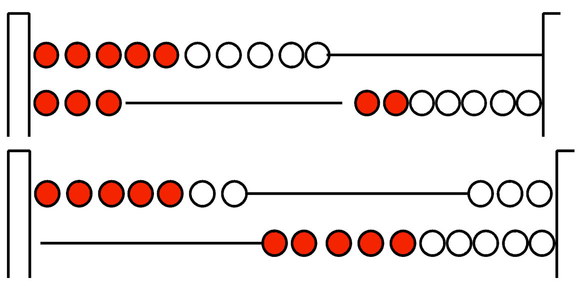
The notation on the left shows the 8 partitioned into 5 and 3 and the 6 partitioned into 5 and 1. The number sentences show that Carlos added the two 5's to get 10 and then added the 4 (from combining the 3 and the 1) to the 10 to get 14. The notation on the right differs in that it shows the two 5's combined to get 10 by drawing a loop around them. The final number sentence shows that the 4 (from combining the 3 and the 1) to 10 to get 14.

As the teacher writes the notation, she will go on to elaborate further, gesturing to the beads and to her notation to link the two and she will get the class involved in the discussion to ensure that they understand the notation and agree that it “shows” how Carlos figured out the problem.

We also note that the same notation works to record the thinking of those children that didn’t just know that 10 and 4 are 14 but counted on from 10 to get 14. But as we’ve commented in the previous sample solution, since our goal is to advance grouping strategies, the teacher will not write anything down to record counting by ones.

It is important to emphasize that for any solution, notation should always reflect the child’s reasoning and not represent a solution method that the teacher has in mind. Finally, remember that we recommend that the teacher introduce notation only after having used tasks of any type for several days.

f*Notation for going down to 10 and beyond grouping strategy.* Now let’s turn to a subtraction example that we discussed earlier. The task was13 people on the bus (shown as 10 on top and 3 on the bottom) and 6 get off. The first solution for was Darius’ solution. Darius moved 3 beads from left to right on the bottom rod (showing that 3 people got off the lower deck of the bus) and moved 3 beads from left to right on the top, showing that 3 people got off the upper deck of the bus. The initial configuration used when the task was posed and the final configuration on Darius’ rack are shown here las a reminder.



Initial configuration Darius’ rack

The discussion focused on how Darius got down to 10 (the 10 beads on the top) and then took 3 off that 10 to get 7. To initiate the use of notation the teacher might say, “I’m going to write down something that shows how Darius thought about it.” Then the teacher can write the number sentences

$$13 - 3 = 10$$
$$10 - 3 = 7$$

while saying, “Darius started with 13. He had 10 on the top and 3 on the bottom. He took away the 3 on the bottom to get down to 10 and then he took 3 away from the 10 and got to 7. That’s how he took the 6 away. He did it in two parts. First he took away 3 to get down to 10 and then he took away 3 more from that 10.” To further emphasize that he took away the 6 in two parts, the teacher might circle the two 3’s in the number sentences and say, “Here’s the 6 that he took away.”

$$\begin{array}{l} 13 - 3 = 10 \\ 10 - 3 = 7 \end{array}$$

Here's an entirely different way to notate Darius' solution.

$$\begin{array}{lll} \multicolumn{3}{c}{13 - 6 =} \\ 10 & \cancel{3} & \\ \underline{-\ 3} & & \\ 7 & & \end{array}$$

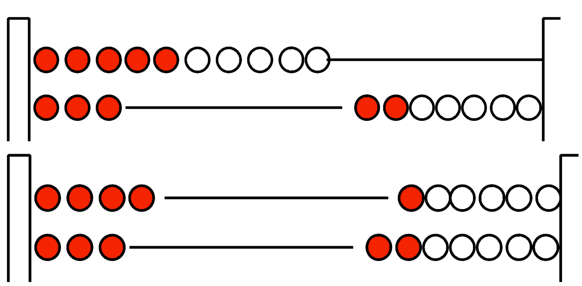
This version of the notation makes explicit that the initial 13 was shown as 10 and 3. The slash through the 3 indicates that the 3 was taken away and then we see that another 3 was taken from the 10 leaving 7.

We have found both forms of notation productive as a way to show this solution method. Most teachers use these ways of notating interchangeably, using one form at some time and the other at other times. Nevertheless, it is important to keep in mind that the notation should reflect the child's reasoning and not represent a solution method that the teacher has in mind.

Here we show these ways of notation side by side for easy comparison.

$$\begin{array}{l} 13 - 3 = 10 \\ 10 - 3 = 7 \end{array} \qquad \begin{array}{l} 13 - 3 = 10 \\ 10 - 3 = 7 \end{array} \qquad \begin{array}{lll} \multicolumn{3}{c}{13 - 6 =} \\ 10 & \cancel{3} & \\ \underline{-\ 3} & & \\ 7 & & \end{array}$$

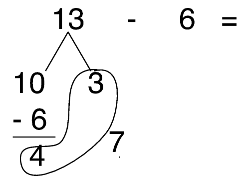
Notation for take all away from 10 grouping strategy. The second solution for the task 13 people on the bus (shown as 10 on top and 3 on the bottom) and 6 get off was Gabriella's solution. Gabriella moved 6 beads on the top rod from left to right, showing that all 6 people got off the upper deck of the bus. The initial configuration when the task was posed and the final configuration on Gabriella's rack are shown here.



Initial configuration

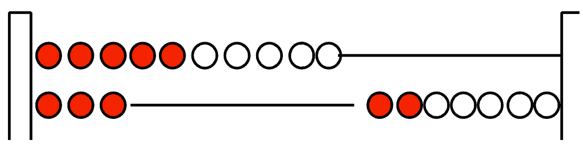
Gabriella's rack

Structuring Numbers Notes for Video Group 2
E. Yackel
The discussion focused on how Gabriella took all 6 off of the 10. The form of notation that we have found to be useful for recording this solution method is shown here.

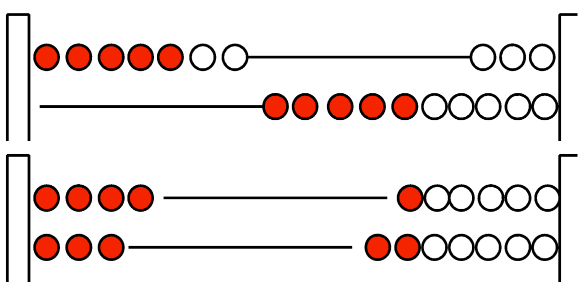


After writing the task numerically, 13 – 6 =, the teacher shows 13 partitioned into 10 and 3, indicating that the 13 was shown as 10 on the top and 3 on the bottom. Next the teacher shows that Gabriella took all 6 away from the 10, leaving 4. The loop around the 4 and the 3 indicates that the 3 beads remained on the bottom rod (no one got off the lower deck of the bus) and 4 remain on the top rod. So the 4 and the 3 together indicate how many people are on the bus now. It's 7.

Before we leave the discussion of notation, let's take a look at the second form of notation we showed for Darius' solution and see it side by side with the notation we've shown here for Gabriella's solution.

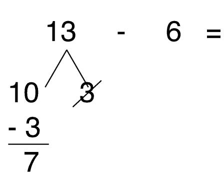


Initial configuration

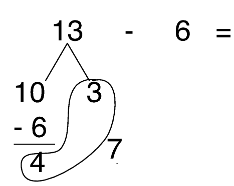


Darius' solution

Gabriella's solution



Notation for Darius' solution

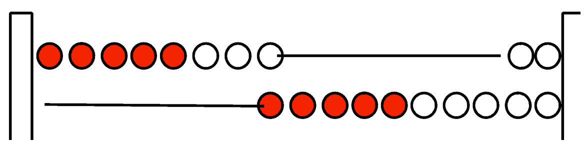


Notation for Gabriella's solution

We include this image to clarify for the teacher how these two forms of notation are alike and how they are different. The rack at the top shows how the task was posed. Thirteen was shown as 10 on the top and 3 on the bottom. In each case the teacher started by writing the task in numerical form, 13 – 6. And in each case she showed the 13 partitioned into 10 and 3. From that point on the two forms of notation differ. The notation on the left shows that the 3 of the 13 was taken away, and then 3 from the 10. The notation on the right shows that all 6 were taken from the 10.

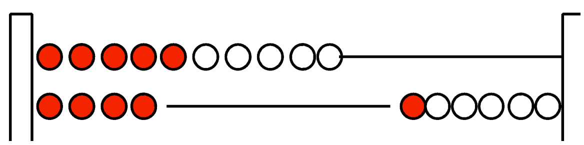
In our experience teachers have found these forms of notation easy to make sense of and use. And consequently so do the students. They provide sufficient detail and yet are not so complex as to be confusing. As a caveat, we caution teachers against showing the 6 as split into 3 and 3 for the notation on the left. Invariably, doing so leaves both the teacher and the students confused. This remark applies to other similar subtraction examples as well. This cautionary note is based on many experiences.

Less common solutions. We have discussed in detail the two most common solutions to the addition example, 8 on the bus 6 more get on where the task was posed with the initial configuration shown here.

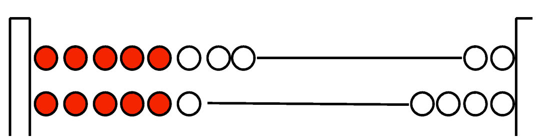


They are:

- 2 get on the top and 4 on the bottom.

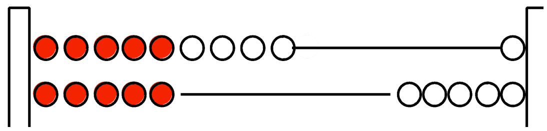


- All 6 get on the bottom.



As we have explained, the first of these solutions gives rise to the filling up the 10 grouping strategy. The second of these gives rise to the combining 5's grouping strategy. Both of these grouping strategies are ones that we know sophisticated children use when reasoning purely numerically. For this reason, we have as an instructional goal that children that would not develop these ways of reasoning spontaneously on their own are given an opportunity to do so, arising from their physical actions with the beads. And the pedagogical notation that we showed is useful for those children to help them advance to eventually reason using these strategies when tasks are posed purely numerically.

Now there is a third possible way to move the beads to complete this task. It is to move 1 bead on the top to the left and 5 beads on the bottom to the left. This is the solution method Brian used.

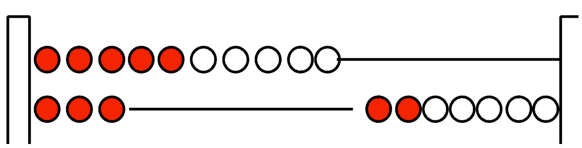


Brian

Even this way of moving the beads gives rise to a resulting configuration that can be easily read. However there is no efficient related grouping strategy. Here's what I mean. When reasoning purely numerically to solve the task 8 + 6, we do not see sophisticated children reason by thinking, "I'm going to take 1 from the 6 and add it to 8 to get 9, and then add on the remaining 5 of the 6 to 9 to get 14." The reason they don't do this is because adding small numbers on to 9 is not easy. The reason the other solutions are common is because each of them involved getting a 10. And 10 plays a special role in our number system. Adding on to 10 results in the related teen number and by this time many children in the class know this.

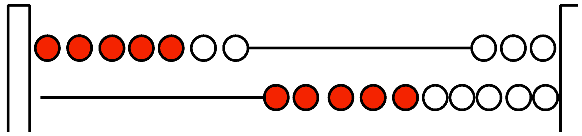
For this reason if a child uses this third way to solve the task, we do not devote time during the discussion to this solution. There is no efficient grouping strategy to relate it to. Of course a student might report that he solved the task in this way and might even explain how he used the color configuration of the beads to figure out how many are on the bus now. But we do not encourage teachers to spend a lot of time getting others in the class involved in discussing this solution and we recommend not using numerical notation to record this solution. Doing so inevitably leads to confusion. The solution is not invalid. It simply does not lead to an efficient grouping strategy. In other words, it does not advance the agenda for which this part of the sequence is designed.

Let's consider less common solutions for the subtraction example, 13 people on the bus, 6 get off. The task was posed using the initial configuration shown here.

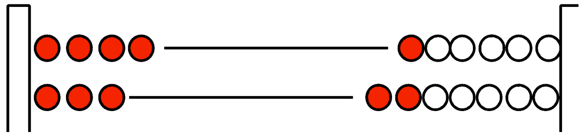


We have already discussed in detail two solutions to this task that are the most common. They are:

- 3 get off the bottom and 3 get off the top.

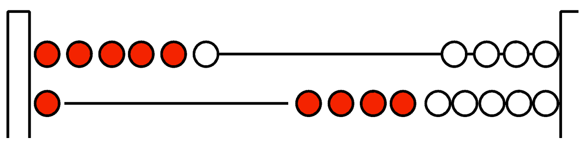


- All 6 get off the top.

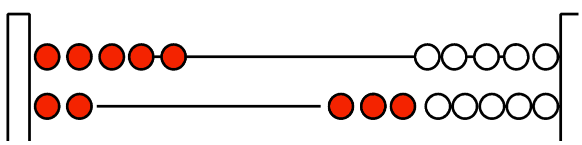


The first of these solutions gives rise to the going down to 10 and beyond grouping strategy. The second of these gives rise to the taking all from 10 grouping strategy. Here again, both of these grouping strategies are ones that we know sophisticated children use when reasoning purely numerically. For this reason we have as an instructional goal that children that would not develop these ways of reasoning spontaneously on their own are given an opportunity to do so arising from their physical actions with the beads. And the pedagogical notation that we showed is useful for those children to help them advance to eventually reason using these strategies when tasks are posed purely numerically.

For this task there are several other ways to move the beads as well. One way is to take 2 off the bottom and 4 off the top.



Another is to take 1 off the bottom and 5 off the top.



But here again, there are no efficient grouping strategies that are related to each of these ways. When solving tasks in purely numerical settings, sophisticated children do not reason to take 1 from the 3 and 5 from the 10. Nor do they reason that they will take 2 from the 3 and 4 from the 10. For this reason, if a child uses one of these ways to solve the task, we do not devote a lot of time during the discussion to the solution. There is no efficient grouping strategy to relate it to. And we recommend not using numerical notation to record these solutions. In our experience, teachers that try to do so inevitably create confusion. Certainly, when called on, a student might report that he solved the task in one of these ways and might go on to explain how he used the color configuration of the beads to figure out how many are on the bus now. But we do not encourage teachers to spend a lot of time getting others in the class involved in discussing these solutions. They are not incorrect solutions. They simply do not contribute to developing efficient grouping strategies. They do not advance the agenda for which this part of the sequence is designed.

Summary Remarks about Notation. We have shown several different ways of notating children's reasoning for both the addition and subtraction examples. The principle that under girds the use of notation is that initially the notation serves as a *record of students' reasoning*. The notation gradually shifts in function to becoming a *means for students' reasoning***.** This shift in the function of notation is subtle and is not the result of teacher explanation. Rather it is the result of student reflection. Therefore the shift takes place gradually and at different times for different students. For this reason it is important for the teacher to carefully think through notation as it will be used extensively over many lessons.

Where did we get this notation? These ways grew out of situations where we were working in classroom with teachers as they used the SN sequence. In some cases the teacher introduced something spontaneously that proved productive. In those cases, we have suggested that same notation to other teachers who confirmed its value. In other cases, we have worked with teachers to try out various forms of notation that can be thought of as recording the children's reasoning. The forms of notation we have shown are those that have met the following criteria:

1. The notation should be productive for advancing children's development of grouping strategies.
2. Children should agree that the notation shows how they reasoned. (It should be a record of their reasoning.)
3. The notation should be clarifying and not confusing. That means it should be relatively concise. Not every detail has to be recorded.
4. The notation should be relatively easy for the teacher to make sense of and use comfortably.

Finally we want to emphasize once again that not every solution should be notated. In particular, the teacher should avoid using notation to try to record solutions that are not linked to efficient grouping strategies.

When notation is used it should serve the purpose of aiding communication. It should help children make sense. If children are struggling to make sense of some notation, there is a problem. Notation should make it easier to understand the reasoning, not harder. And it should always reflect the child's reasoning and not represent a solution method the teacher has in mind. It is important that when planning the lesson, teachers anticipate solutions children will give and plan which solutions they will record and how they will do so. We also recommend that teachers not notate solutions for the missing addend and missing subtrahend tasks we described as part of the "reasoning without moving the beads" activity but limit notating to addition and subtraction tasks.

WHEN to begin recording children's solutions. In explaining the purpose for recording children's solutions we have said that numerical notation provides a link between mathematical symbols and the numerical relationships indicated by the grouping strategies that emerged from children's actions with the beads on the Arithmetic Rack. In particular, we are interested in the strategies for addition and subtraction tasks that involve numbers to 20, with an emphasis on teen numbers. The tasks in Part II of the SN sequence are precisely the types of tasks involved in developing these number relationships. For this reason we recommend that throughout this portion of the sequence, beginning with the "reasoning by moving the beads" activity and continuing throughout, teachers use the types of notation we have shown to record children's solutions. Even so, as we have noted when presenting the examples, we suggest that teachers use tasks for several days before introducing notation. The reason for this suggestion is that it is helpful for students to focus for several days entirely on thinking through the numerical relationships (that is, the grouping strategies) that are implicit in the way the beads are moved. After two or three days where that is the entire focus, it is appropriate for the teacher to begin using notation. By that time, a number of students in the class will be starting to think and reason numerically.

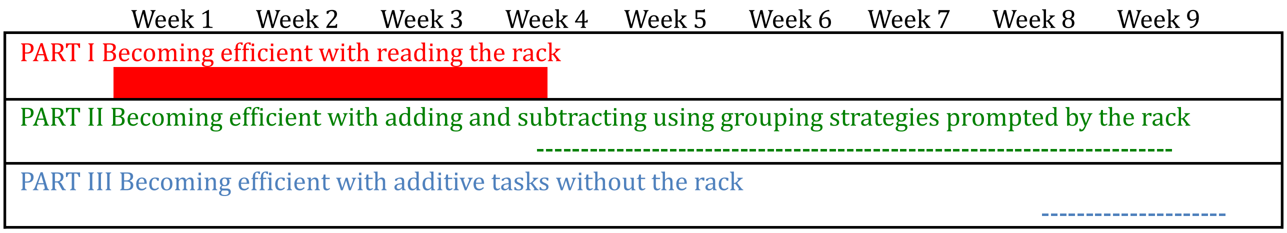
The repeated and persistent use of notation is crucial to children's developing meaning for numerical symbols. In this way, rather than being introduced in a top down fashion, numerical symbols come in as a means to express reasoning the children have already engaged in. The symbols are a record of their already completed reasoning activity. Later, when children encounter numerical symbolism on its own, they will have a basis for making sense of it.

I want to make one additional remark about notation here. The activities in this part of the sequence involve both addition and subtraction tasks that are intermingled throughout the lessons. That is, teachers do not use only addition tasks and then later introduce subtraction tasks. Rather they are used in conjunction with each other, as we have illustrated in some of the suggested task sequences. Even so, teachers we have worked with have found that it takes them some time to become proficient with using the suggested notation. For that reason teachers typically introduce notation for addition tasks several days before they introduce notation for subtraction tasks. Doing so, not only gives teachers an opportunity to become comfortable with one format of notation before using another, but also gives students an opportunity to do so as well. This rationale is simply pragmatic.

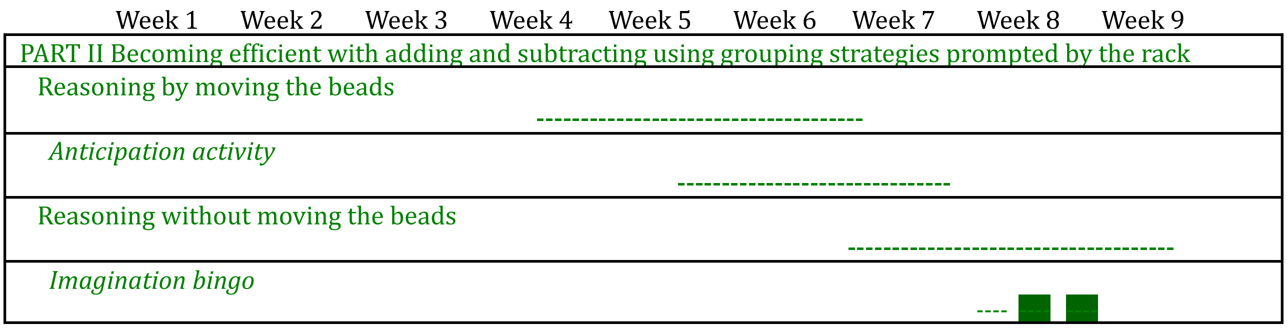
Timeline for Structuring Numbers Instructional Sequence

In these notes we have given detailed information about the second part of the Structuring Numbers instructional sequence. Here we show a timeline for the entire sequence that shows where Part II fits in. Typically teachers devote about three and one half weeks to Part I of the sequence before moving to Part II. As an aside we note that it is not necessary to continue with activities from Part I while also beginning Part II because reading the rack is an integral part of each of the activities within Part II. Children that may not yet be completely proficient with reading the rack efficiently continue to have opportunities to do so as they engage in the activities within Part II. The timeline shows that Part II continues almost to the end of the entire sequence.

Structuring Numbers Notes for Video Group 2
E. Yackel



Below we show the detailed timeline we show below for Part II.



This detailed timeline indicates that the “reasoning by moving the beads” activity is used for about two and one half weeks before it gives way to the “reasoning without moving the beads” activity, which is itself used for about two and one half weeks. The activity we called the “anticipation activity” serves as a transition between these two activities. It is first used after several lessons of “reasoning by moving the beads” and is then combined with that activity to comprise a lesson. When the shift is made to reasoning without moving the beads, the anticipation activity is still used as part of the lesson for some time. This overlap is deliberate. When students begin to reason by moving the beads some of them very quickly begin to think about which beads they would move even before they do so. The anticipation activity capitalizes on this advance by posing tasks where student begin by moving beads on their racks, but then are asked to try to complete the action and figure out the total mentally, without completing the action with the beads. In this way the anticipation activity provides important conceptual challenges for these students that help to advance their learning. At the same time, those students that are not yet able to complete the task mentally have access to their racks so they can complete the task by moving the beads on the rack or at least by touching the beads on the rack. In this way the teacher ensures that every student participates in every instructional task posed and is able to do so in a personally meaningful way. Imagination bingo comes in toward the end of Part II and is an activity that provides variety in the format and also contributes to solidifying children’s conceptual understanding of the relevant number relationships for numbers to 20.

Part II forms the heart of the Structuring Numbers sequence. We encourage teachers to use their judgment about the total amount of time to devote to this part. At the same time we remind teachers to keep moving forward. These instructional activities are designed so that children continue to make conceptual advances even as they move to more conceptually challenging activities.

Next Video Groups and Notes

There are two additional sets of notes, one to accompany each of the video groups:

- Structuring Numbers III
- Teacher’s role in implementing the SN sequence

The first of these describes Part III of the Structuring Numbers sequence in detail.
The second is a set of notes to accompany the video group that focuses on the teacher’s role in successfully implementing this sequence. It is devoted to two main issues. One relates to planning lessons, including selecting which activities to combine to comprise a lesson and, for any specific activity, selecting which tasks to pose to enhance the learning opportunities for students. The second issue is how to orchestrate productive class discussions. Examples are used to illustrate both of these issues.

The fourth video group can be viewed meaningfully at any time. It is not necessary to first view all three of the video groups that describe the details of the instructional activities. Likewise, the set of accompanying notes will be meaningful to the teacher at any time.